YIELD CURVE FITTING 1.07

User Guide

CONTENT

1. INTRODUCTION

The current actuarial practice often requires calculation of the market value of assets as well as liabilities. Relevant market YC determined from a portfolio of a given pool of assets is the essential part of all of these calculations. Here you receive a tool which should cover your needs in this particular area. The tool has been created by experienced actuaries in close cooperation with professional IT colleagues. We believe you will enjoy working with this tool. If you have any comments, ideas of improvements or other remarks, please contact us through our web site.

2. RUNNING THE APPLICATION

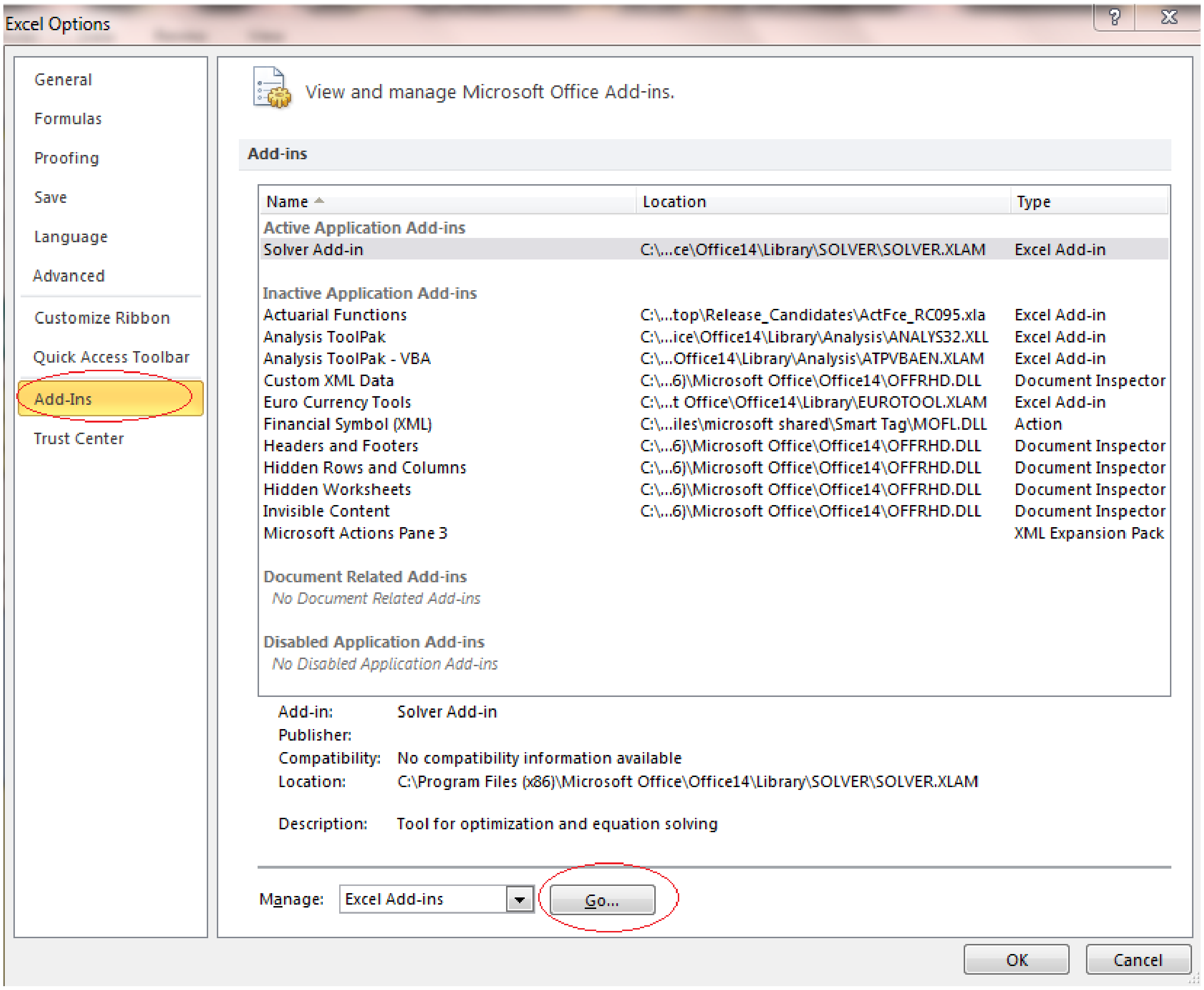
MS Excel environment has been use for creation of this tool. The MS Excel must meet these requirements, or it may not be able to successfully working:

- Add-Ins: Analysis Tool Pack – VBA, Solver;
- Macros enabled.

Also be aware the fact if you use the Excel Starter edition, the application does not run correctly.

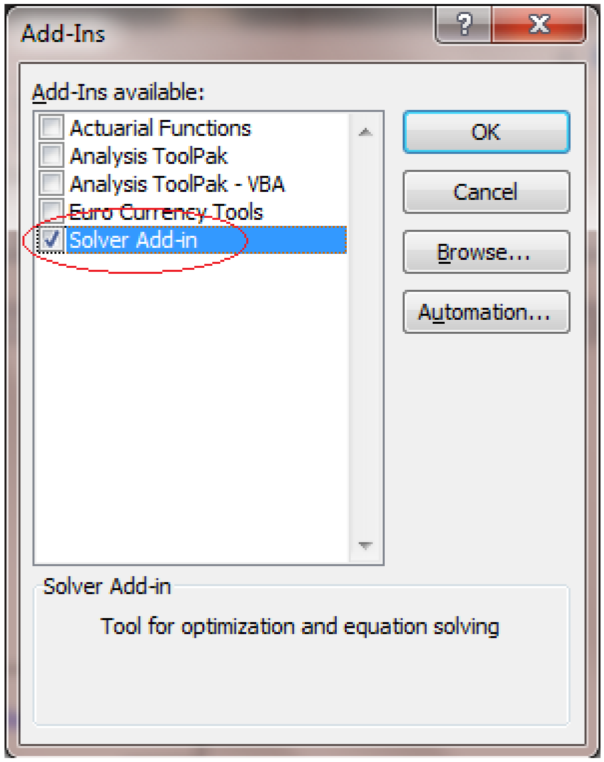
Before you open the application it is necessary to add the Solver to your Excel file.

Open the blank Excel file and in the main menu click on the “File” and “Options”. The window Excel options is displayed on your screen. Now select the option “Add-ins” from the left menu. Down on the page click on the button “Go …”.



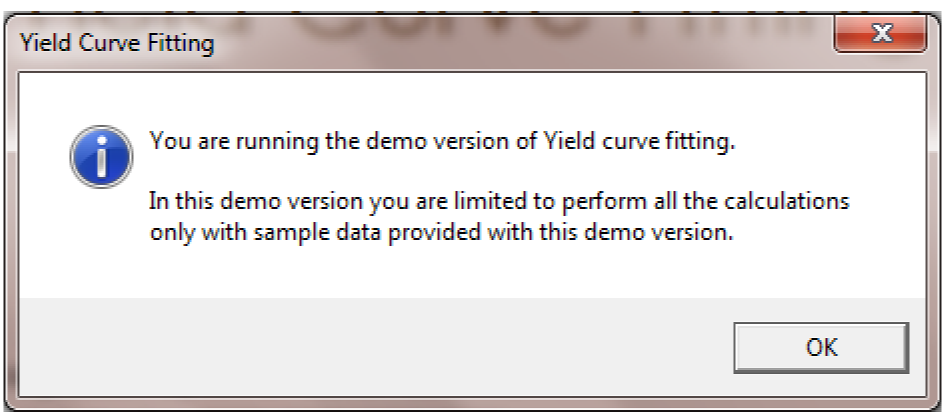
To run application correctly, check the checkbox “Solver Add-in” and click “OK”. You need to do this step only once when first opening the application.

Once you are done with this step, click on the option “Open” in the main menu which let you choose the application saved on your disc.

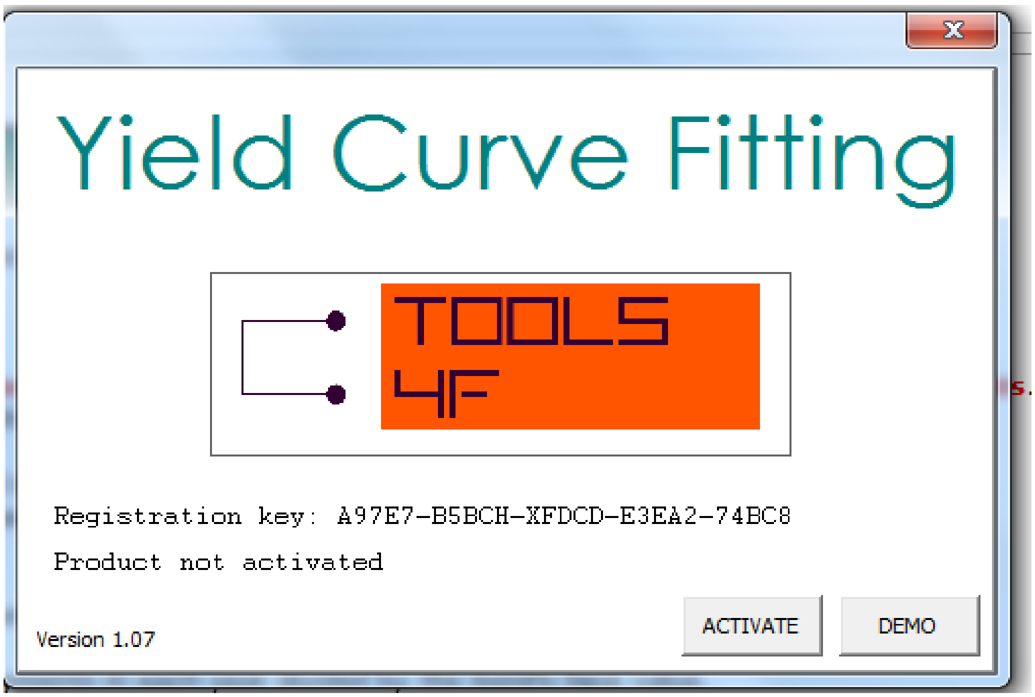


3. DEMO VERSION AND ACTIVATION

After opening the application, you will be informed about running the demo version. Now you can only run the demo version until you insert the valid product key. In the demo version you can only use the sample data provided inside the demo. Click “OK” to continue.

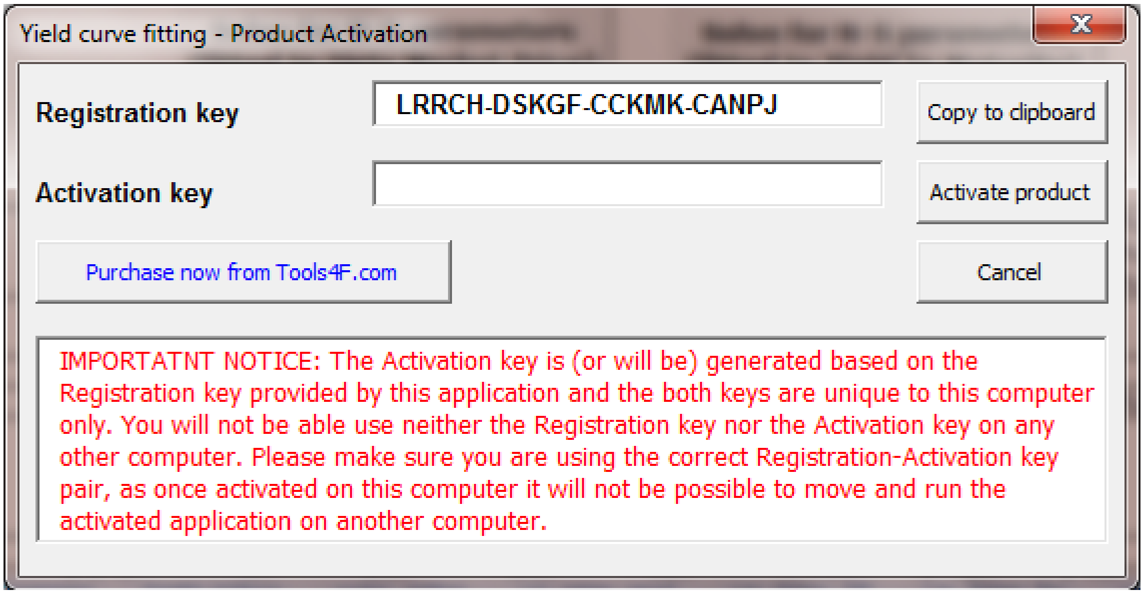


Now only the demo version is active. If you want to run the full version of the application, click on the button “Activate” otherwise continue in the demo version by clicking on the button “Demo”.



To get the Activation key, go to the web site www.tools4f.com and purchase the application. Once the license is ordered and paid, to your e-mail you will be sent the Activation key. After receiving the Activation key, copy it to the box “Activation key” and you can activate the full version of this application with the button “Activate product”.

NOTE: The Activation key is generated based on the Registration key provided by this application and the both keys are unique to this computer only. You will not be able use neither the Registration key nor the Activation key on any other computer. Please make sure you are using the correct Registration - Activation key pair, as once activated on your computer it will not be possible to move and run the activated application on another computer.



From now on, you can use all functions of the application.

4. Using the application

5. Inputs

The first thing you should do is to fill in the input data. That means the information about the financial instruments which form your portfolio, necessary to determine the YC. These might be sets of bonds or interest rate swaps (IRS). The following information must be filled:

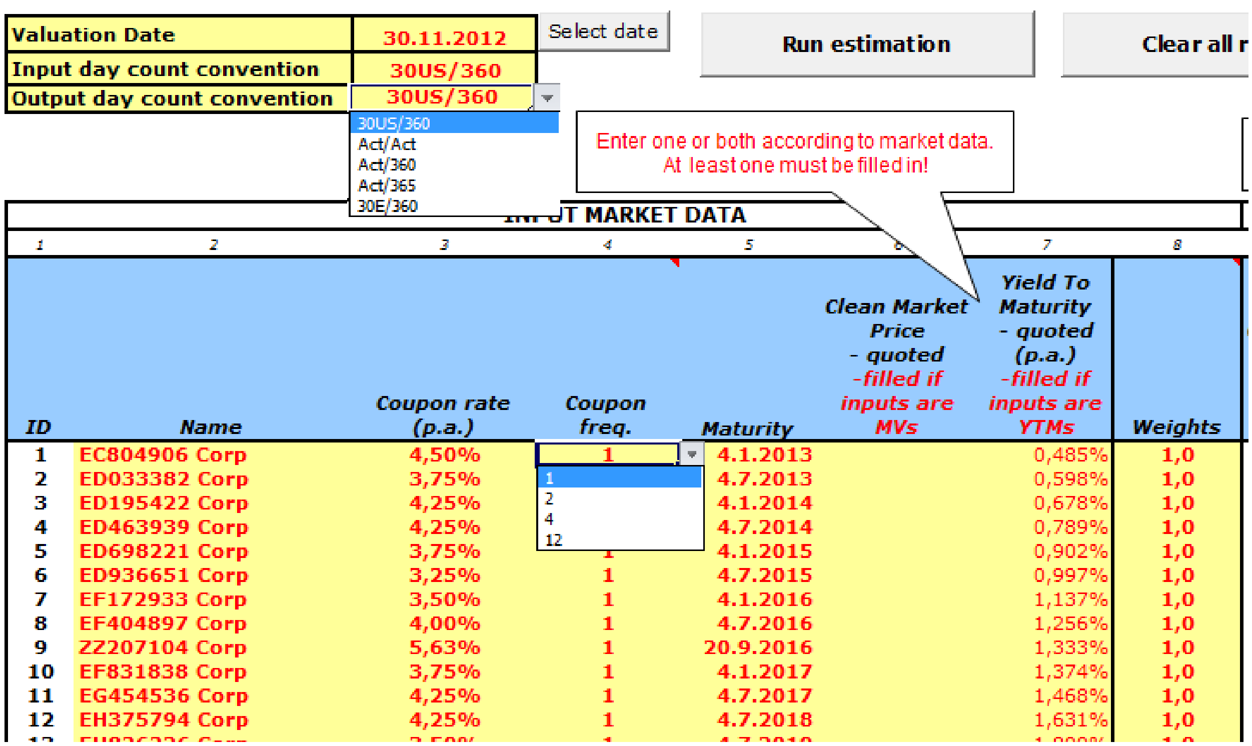
- **Valuation date** (The button “Select date” allows you to select the date related to the market data)
- **Name** (of the bond or IRS , for the identification)
- **Coupon rate** (p.a., the total amount of coupons paid per year and divided by the bond's face value)
- **Coupon frequency** (1 – annually, 2 – semi-annually, 4 - quarterly and 12 - monthly)
- **Maturity** (the date)
- **Clean Market Price** (The price of a bond excluding any interest that has accrued since issue or the most recent coupon payment.)

- **Or Yield to Maturity** (The internal rate of return (IRR, overall interest rate) earned by an investor who buys the bond today at the market price, assuming the bond will be held until maturity, and that all coupon and principal payments will be made on schedule.) This yield to maturity corresponds to the clean market price.

- **Weights** (They are used to find parameters of the relevant yield curve fitting methods using numerical method of weighted least squares. This may express the degree of your confidence in the market data of the financial instrument.)

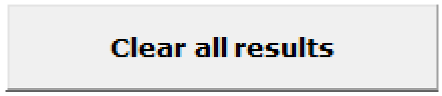
NOTE: The maximum number of instruments is 100 and maximum time to maturity is 50 years.

After this step, you have to choose one of the five input time conventions "Input day count convention" which belongs to the entire input set of instruments (to know more about time conventions and calculation of the time interval, see the chapter called Technical documentation). You can select 30US/360, Act/Act, Act/360, Act/365 or 30E/360. You are required to select "Output day count convention" for resulting yield curves. The same values can be selected as in the previous step: 30US/360, Act/Act, Act/360, Act/365 or 30E/360 (see the explanation of time convention in the chapter called Technical documentation).



ID	Name	Coupon rate (p.a.)	Coupon freq.	Maturity	Clean Market Price - quoted -filled if inputs are MVs	Yield To Maturity - quoted (p.a.) -filled if inputs are YTMs	Weights
1	EC804906 Corp	4,50%	1	4.1.2013		0,485%	1,0
2	ED033382 Corp	3,75%		4.7.2013		0,598%	1,0
3	ED195422 Corp	4,25%		4.1.2014		0,678%	1,0
4	ED463939 Corp	4,25%		4.7.2014		0,789%	1,0
5	ED698221 Corp	3,75%		4.1.2015		0,902%	1,0
6	ED936651 Corp	3,25%	1	4.7.2015		0,997%	1,0
7	EF172933 Corp	3,50%	1	4.1.2016		1,137%	1,0
8	EF404897 Corp	4,00%	1	4.7.2016		1,256%	1,0
9	ZZ207104 Corp	5,63%	1	20.9.2016		1,333%	1,0
10	EF831838 Corp	3,75%	1	4.1.2017		1,374%	1,0
11	EG454536 Corp	4,25%	1	4.7.2017		1,468%	1,0
12	EH375794 Corp	4,25%	1	4.7.2018		1,631%	1,0

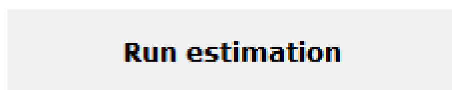
There is the *Clear all results* button in the <Inputs> sheet. You can use it to reset all output yield curves (both annual and monthly), and also to erase all loaded and calculated values from each sheet.

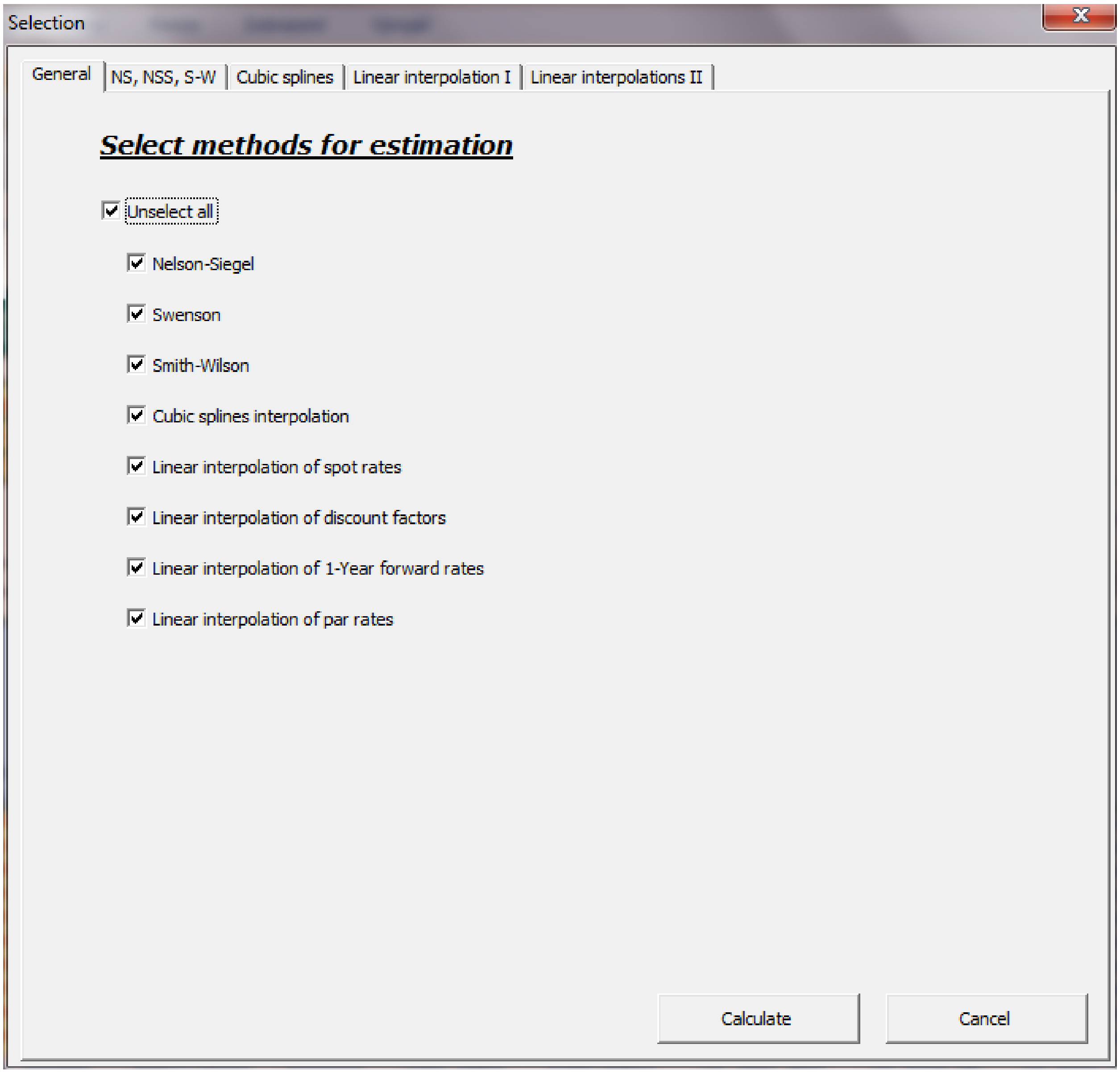


6. SOLVING THROUGH THE USER FORM

Calculation through the user form is recommended.

As the necessary inputs are specified in the <Input> sheet, the specific methods and parameters for the yield curve construction can be set by pressing the “Run estimation” button, which opens the form below. It is very easy to set up all the parameters through the user form.





All the methods of yield curve construction will appear in the first tab named General at the top of the menu. There is a selection of eight possible methods:

- Nelson-Siegel
- Swenson
- Smith-Wilson
- Cubic splines interpolation
- Linear interpolation of spot rates
- Linear interpolation of discount factors

- Linear interpolation of 1-Year forward rates
- Linear interpolation of par rates

You can use the “Select all/Unselect all” checkbox for faster selection. To set specific parameters for each of the individual methods, click on the corresponding tab name located at the top of the menu.

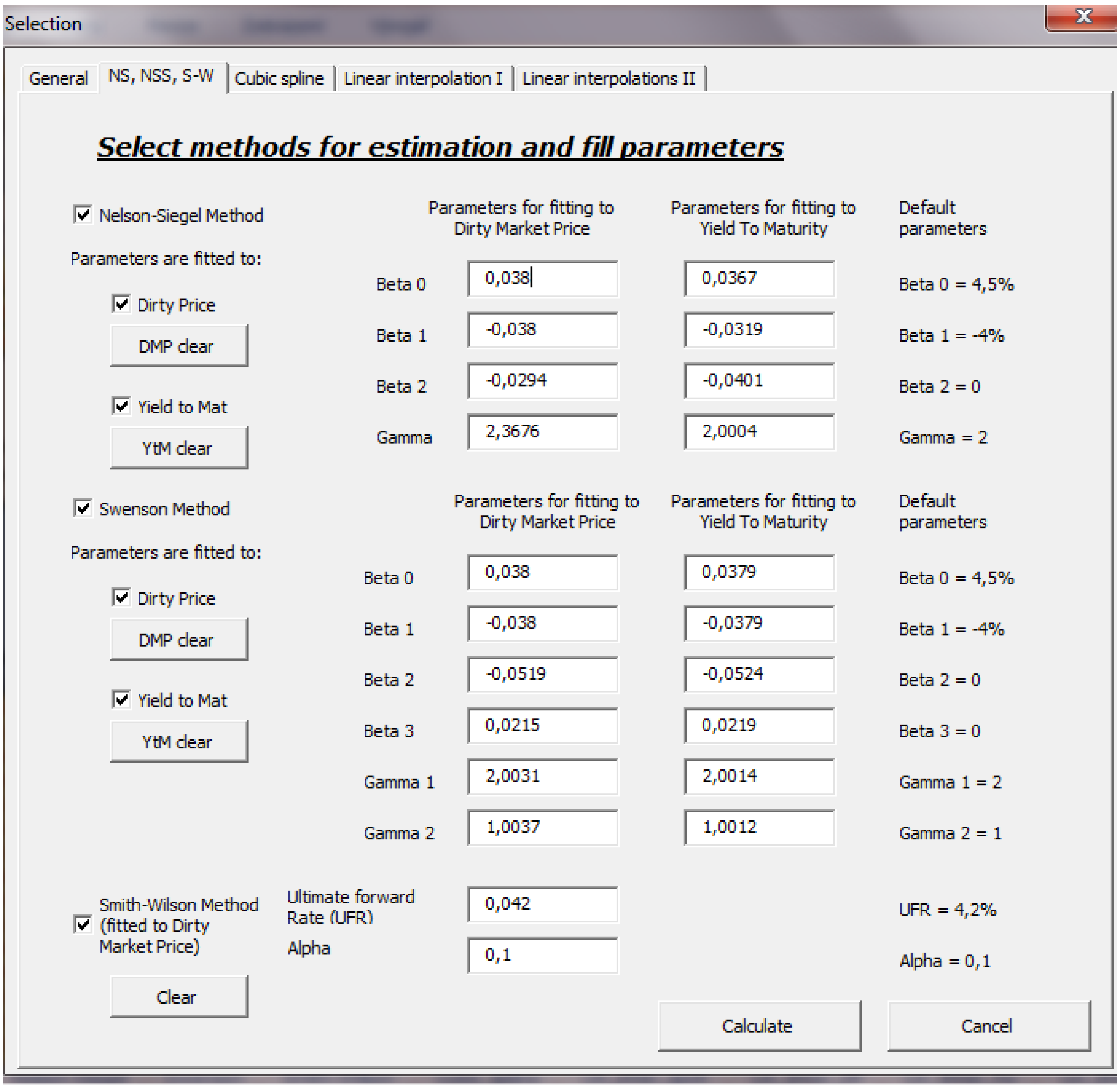
NS, NSS and S-W tab

Generally there are two options for fitting - DMP = Dirty Market Price or YtM = Yield To Maturity. These two fitting options could be set for the following methods: Nelson-Siegel, Swenson, Linear interpolation spot rates, discount factors, forward rates and par rates. For Smith-Wilson method and Cubic splines interpolation use fitting to dirty market price only.

You also have a possibility to delete the input boxes for each set of input parameters using the “DPM clean”, “YtM clean”, “Clean” or “Time clean” buttons in the user form. This could be very useful in case the input boxes contain the parameters loaded from the last configuration.

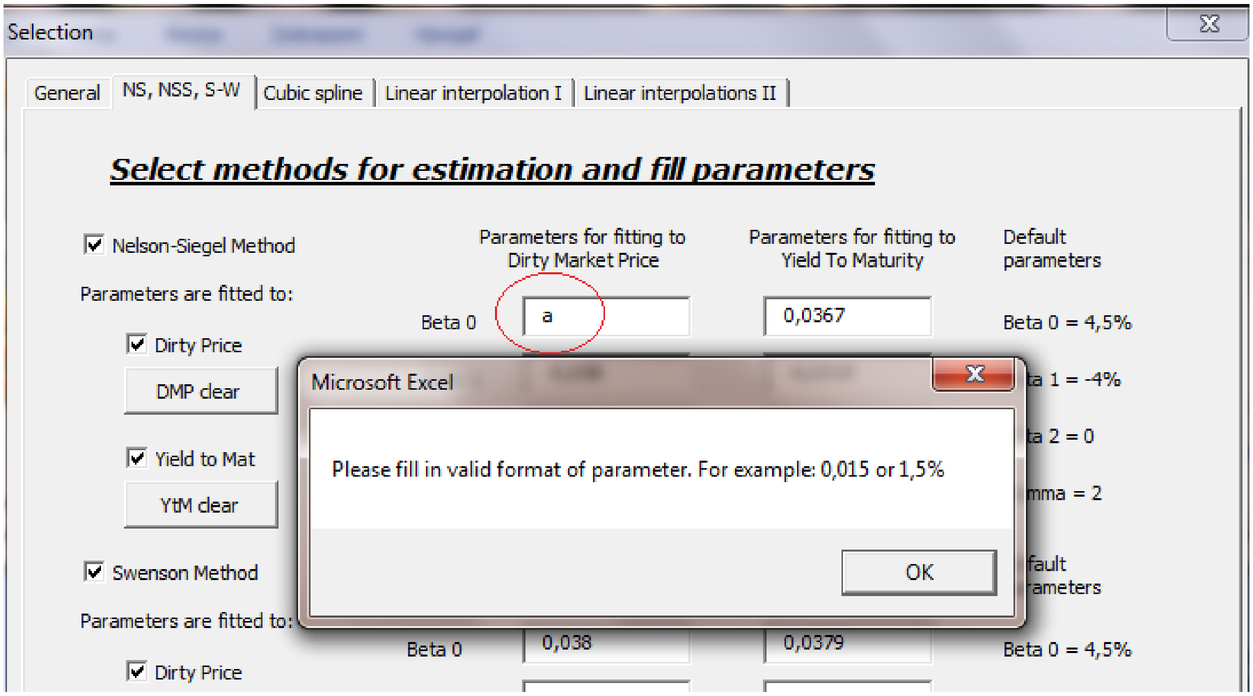
The buttons “… default” allow you to load the default value of parameters (if the boxes are empty).

Recommended parameters mean the “standard” start values of input parameters (inspired by related literature).



Entering all desired parameter values is validated in order to avoid possible wrong values. If wrong values are entered into the box, e.g. text string, warning notification will appear (as shown in the picture below). If you are not sure about the right values, you can load the default values by using the “DMP (YtM) clean/ DMP (YtM) default” button.

NOTE: The default parameters are the ones inspired by literature, however, other starting points may result in a better fit.



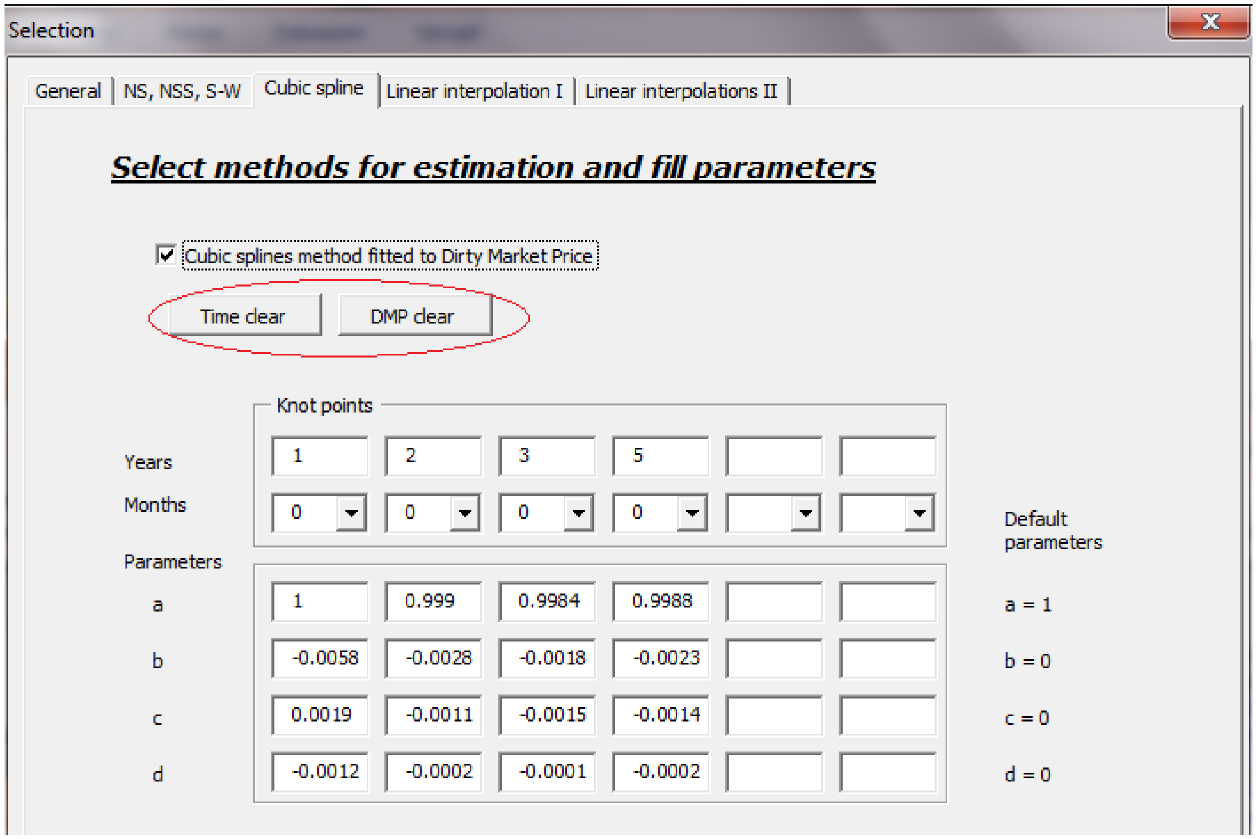
The rules above apply to tabs NS, NSS, S-W.

Cubic splines and linear interpolation

In these tabs, the methods **require the time parameter** - Cubic splines, Linear interpolation I and Linear interpolations II (for more information see the Technical documentation chapter).

Cubic splines method could only be fit to DMP as it is shown in the picture below. The buttons "…clean" and "…default" are also available there.

If you need more information about the knot points or cubic splines method, see the Technical documentation chapter.

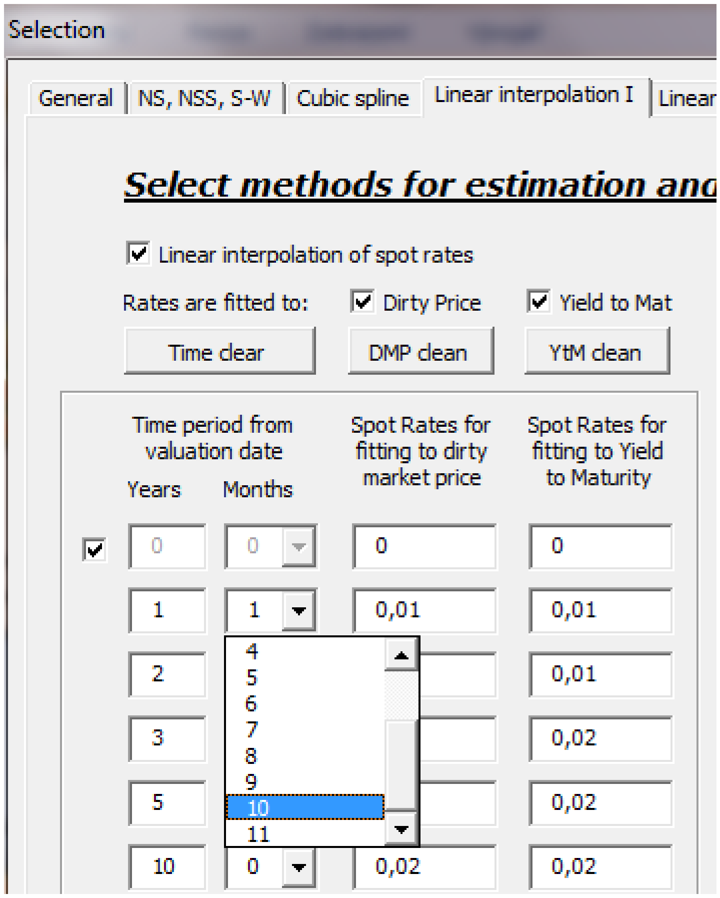


The entering of parameters for linear interpolation will be shown on the method of linear interpolation of spot rates as an example.

As you can seen in the picture below, the first time parameters are disabled because their value must always be the same. 0 years and 0 months signify the beginning from which the rates are linearly interpolated (for more information see the Technical documentation).

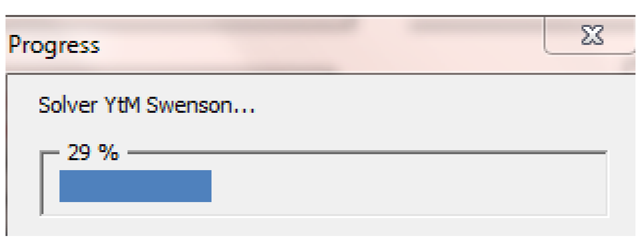
The checkbox placed on the left next to these allows you to consider the rates as variables at this time (if checked) or as unchanging constant while searching the solution.

In the second row, you can possibly specify the time interval from the valuation date. In this example, the zero rate for 1 year and 10 months is 1 %. The actual number of days between valuation date and valuation date + 1 year and 10 months is subsequently converted according to the specified input conventions in the calculation procedure.



The principle of entering the parameters is the same for all other methods: Linear interpolation of discount factors, linear interpolation of 1-Year forward rates and linear interpolation of par rates.

Once the parameters are filled-in, the “Calculate” button allows you to start the calculation process. The progress bar will appear and indicate the progress and the state in which the process is found.

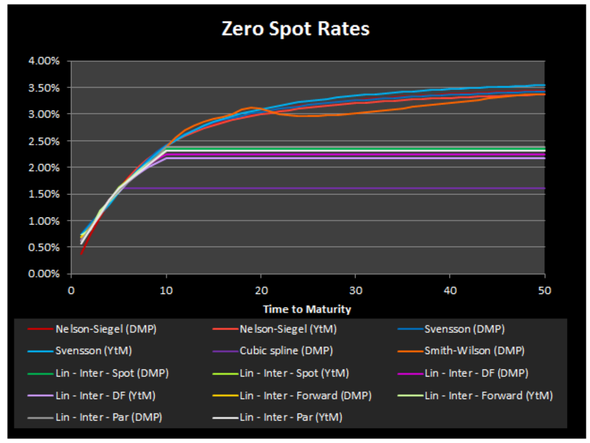


Once all the calculations are complete, the progress bar disappears and the resulting yield curves are available in the <Outputs> and <Outputs monthly> sheets. There are interest rates in the output convention which were selected in the <Inputs> sheet.

In the <Outputs> sheet, annual interest rates for each method are displayed – zero (spot) rates, 1-year forward rates, continuously compounded zero rates and par rates. Corresponding graphs are an integral part of the <Outputs> and they are placed in the lower part of the sheet.

Zero (Spot) Rates

Years	1	2	3
Nelson-Siegel (DMP)	0,38%	0,74%	1,06%
Nelson-Siegel (YtM)	0,64%	0,85%	1,10%
Svensson (DMP)	0,74%	0,96%	1,13%
Svensson (YtM)	0,73%	0,92%	1,11%
Cubic spline (DMP)	0,66%	0,88%	1,13%
Smith-Wilson (DMP)	0,67%	0,89%	1,13%
Lin - Inter - Spot (DMP)	0,67%	0,86%	1,15%
Lin - Inter - Spot (YtM)	0,69%	0,85%	1,12%
Lin - Inter - DF (DMP)	0,64%	0,85%	1,11%
Lin - Inter - DF (YtM)	0,63%	0,83%	1,15%
Lin - Inter - Forward (DMP)	0,70%	0,87%	1,13%
Lin - Inter - Forward (YtM)	0,73%	0,82%	1,18%
Lin - Inter - Par (DMP)	0,62%	0,86%	1,10%
Lin - Inter - Par (YtM)	0,57%	0,86%	1,11%



The <Outputs monthly> sheet is another possibility of the output, which shows the curves of monthly interest rates. This applies to zero (spot) rates, 1-year forward rates, 1-month forward rates and continuously compounded zero rates. It is obvious the <Outputs monthly> also contains the corresponding graphs.

Zero (Spot) Rates

Years	Month	Nelson-Siegel (DMP)	Nelson-Siegel (YtM)	Svensson (DMP)	Svensson (YtM)	Cubic spline (DMP)
1	1	0,03%	0,55%	0,11%	0,42%	0,64%
	2	0,06%	0,55%	0,21%	0,46%	0,63%
	3	0,10%	0,55%	0,30%	0,50%	0,63%
	4	0,13%	0,56%	0,37%	0,53%	0,62%
	5	0,16%	0,56%	0,44%	0,56%	0,62%
	6	0,19%	0,57%	0,50%	0,59%	0,62%
	7	0,22%	0,58%	0,55%	0,62%	0,63%
	8	0,26%	0,59%	0,60%	0,65%	0,63%
	9	0,29%	0,60%	0,64%	0,67%	0,64%
	10	0,32%	0,61%	0,68%	0,69%	0,64%
	11	0,35%	0,63%	0,71%	0,71%	0,65%
	12	0,38%	0,64%	0,74%	0,73%	0,66%
2	13	0,41%	0,65%	0,77%	0,75%	0,68%
	14	0,44%	0,67%	0,79%	0,76%	0,69%
	15	0,47%	0,69%	0,82%	0,78%	0,71%
	16	0,50%	0,70%	0,84%	0,80%	0,73%

7. Solving through each sheet

You can also run the calculation of individual methods of the yield curve construction from each of the sheets.

Nelso-Siegel, Swenson and Smith-Wilson sheets

In the first step you should specify the parameters – e.g. beta0, beta1, beta2 and gamma in the sheet <Nelson-Siegel>. The minimization cell is a cell in which there is the sum of squared differences of model and market prices, or model and market yield to maturity. The optimal solution is found by minimization of this cell.

The variables from the <Inputs> sheet and parameters set are loaded by pressing "Entry parameters". The variables from the <Inputs> sheet will be shown in the lower part of the active sheet.

Use the "Clear all" button to delete all calculated values and to load input parameters displayed in this active sheet. This will delete all variables except for the parameters in the upper part of the sheet.

You have a possibility to find the solution for fitting a method separately. Press the "Solve for N-S parameters (fitted to Dirty Market Price)" button to find the solution by fitting to Dirty Market Price. Likewise, press the "Solve for N-S parameters (fitted to Yield to Maturity)" button to find the solution for fitting to Yield to Maturity.

The "Solve for …" buttons allow you to start the fitting procedures with upgraded parameters again.

NOTE: It is highly recommended to load the variables and entry parameters again (by clicking on the "Load parameters" button) with each update.

The <Smith-Wilson> sheet contains only the "Solve for Smith-Wilson Method (fitted to Dirty Market Price)" and "Clear all" buttons. Loading of the input parameters is not necessary in this case. This method creates an exact model of dirty market price. Explanation is to be found in the Technical documentation chapter.

Cubic splines sheet

In the Cubic splines sheet, there is just one option of fitting – Dirty Market Price. The idea of range of cells named Conditions is explained in the Technical documentation chapter.

Linear interpolation sheets

The work with time parameters is a little bit different in these sheets. The first time parameter marked in blue represents the overnight rate. In the <Lin_Inter_fw> sheet, there is also the second parameter marked in blue, which means 1-year forward rate from 0 years to 1 year, i.e. 1 year zero rate. These time parameters cannot be changed.

The zero interest rate in the time 0 is the beginning of the interpolation and, as in the user form, there is also the option to consider this rate as a variable if “Enable to solve” is checked or as an unchanging constant if the “Enable to solve” is not checked.

If you want to use months as a parameter, the notation has to have the structure “=1+10/12“ in the cell, as you can see in the picture below.

Note that the bonds with maturity higher than the last value of the time parameters are naturally not relevant for this analysis.

NOTE: Time parameters must be sorted in ascending order.

Parameters (time)	Fitted to Dirty Price	Fitted to Yield to Maturity
0	100,00%	100,00%
=1+10/12	99,52%	99,61%
2	98,78%	98,70%
3	97,60%	97,63%
5	93,53%	93,38%
10	80,92%	81,11%
	0,10%	0,10%
	0,10%	0,10%
	0,10%	0,10%
	0,10%	0,10%
	0,10%	0,10%
minimization	0,0000	0,0000

☐ Enable to solve

You can also see the annual output rates of each method in the corresponding sheets (as shown in the pictures below).

Outputs: Fitted to Dirty Market Price

Year	1	2	3	4
Zero (Spot) Rates	0,70%	0,87%	1,13%	1,39%
1-Year Forward Rates	0,70%	1,04%	1,65%	2,17%

Outputs: Par Rate (DP)

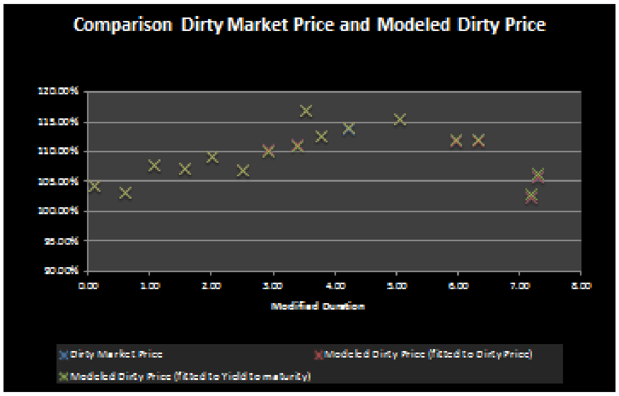
Year	1	2	3	4
Par Rate	0,70%	0,87%	1,12%	1,38%

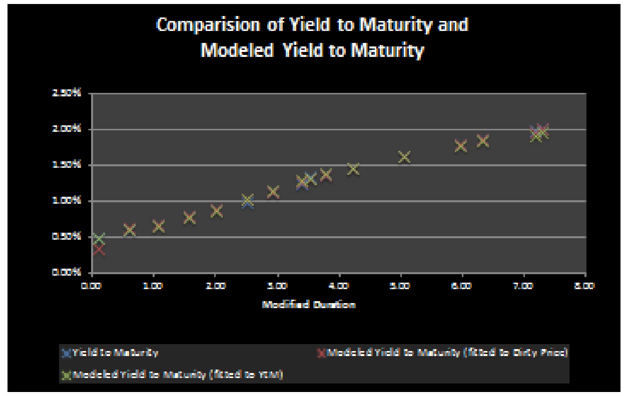
Outputs: Fitted to Yield to Maturity

Year	1	2	3	4
Zero (Spot) Rates	0,73%	0,82%	1,18%	1,37%
1-Year Forward Rates	0,73%	0,92%	1,91%	1,91%

Outputs: Par Rate (YtM)

Year	1	2	3	4
Par Rate	0,73%	0,82%	1,18%	1,36%





8. Technical documentation

9. Nelson-Siegel model

Nelson-Siegel model is described by continuous spot rate at given time *t*, where *t* is positive.

$$r(t) = \beta_0 + \beta_1 * \frac{\left(1 - e^{-t/\gamma}\right)}{t/\gamma} + \beta_2 * \left(\frac{\left(1 - e^{-t/\gamma}\right)}{t/\gamma} - e^{-t/\gamma} \right) \qquad (1)$$

where

$\beta_0, \beta_1, \beta_2, \gamma$ are parameters that determine the shape of the yield curve.

The value of β_0>0, determines an asymptote of the zero coupon yield curve.

$\beta_0 + \beta_1$> 0 is interpreted as the instantaneous spot rate.

This parameter β_1> 0 could be interpreted as a curvature of the function eventually as a difference between long-term and short-term spot rate.

Formula (1) can be also rewritten as follows:

$$r(t) = \beta_0 * h_0(t) + \beta_1 * h_1(t) + \beta_2 * h_2(t) \qquad (2)$$

where

$$h_0(t) = 1$$

$$h_1(t) = \frac{\left(1 - e^{-t/\gamma}\right)}{t/\gamma}$$

$$h_2(t) = \left(\frac{\left(1 - e^{-t/\gamma}\right)}{t/\gamma} - e^{-t/\gamma} \right)$$

Thus the spot curve can be seen as a linear combination of three component functions which represent a flat, sloped and humped curve. Once the function of continuously compounded zero (spot) rates is known, zero (spot) rates (annually compounded, p.a.), forward rates, par rates and discount factors at each *t*, especially discount factors at each time between cash flow payment dates and valuation date specified from input financial instruments can be calculated. Scalar product of particular instrument cash flows and discount factors gives modeled dirty price. Minimizing the sum of squared differences between these modeled dirty prices and Dirty Market Prices (DMP) results in estimators of parameters $\beta_0, \beta_1, \beta_2, \gamma$ fitted to DMP. Modeled dirty market price minus accrued interest gives modeled clean market price and then also modeled Yield to Maturity (YtM), which corresponds to the Clean Market Price. Minimizing the sum of squared differences between modeled YtM and market YtM, we get estimators of parameters $\beta_0, \beta_1, \beta_2, \gamma$ fitted to YtM.

Default starting values of Nelson-Siegel parameters are in the following table 1.

β0	4,5 %
β1	-4 %
β2	0
γ	2

Table1: Default starting values of Nelson-Siegel parameters

For more information you can read [4], [5] and [6].

10. Swenson Model

Swenson´s model is a kind of extension of the Nelson-Siegel model following an idea of adding an additional hump-shaped element ($h_4(t)$) in order to better fit the yield curve shapes. (Particularly this additional element is needed for fitting long-term instruments.)

Swenson model is defined by continuous spot rate function:

$$r(t) = \beta_0 * h_0(t) + \beta_1 * h_1(t) + \beta_2 * h_2(t) + \beta_3 * h_3(t) \qquad (3)$$

where

$\beta_0, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2$ are parameters that determine the shape of the yield curve and

$$h_0(t) = 1$$

$$h_1(t) = \frac{\left(1 - e^{-t/\gamma_1}\right)}{t/\gamma_1}$$

$$h_2(t) = \left(\frac{\left(1 - e^{-t/\gamma_1}\right)}{t/\gamma_1} - e^{-t/\gamma_1} \right)$$

$$h_3(t) = \left(\frac{\left(1 - e^{-t/\gamma_2}\right)}{t/\gamma_2} - e^{-t/\gamma 2} \right)$$

Thus, as it can be seen, the spot curve is a linear combination of four element shapes – a flat, sloped and two humped curves. Once the function of continuously compounded zero rates is given, spot rates (discretely compounded, p.a.), forward rates, par rates and discount factors at each time can be calculated. Thus discount factors at each time between cash flow payment dates and valuation date specified from input financial instruments are obtained. Scalar product of particular instrument cash flows and discount factors gives modeled dirty price. Minimizing the sum of squared differences between these modeled dirty prices and Dirty Market Prices (DMP) results in estimators of parameters $\beta_0, \beta_1, \beta_2, \gamma_1, \gamma_2$ fitted to DMP. Modeled dirty market price minus accrued interest gives us modeled clean market price and then also modeled Yield to Maturity (YtM), which corresponds to Clean Market Price. Minimizing the sum of squared differences between modeled YtM and market YtM gives estimators of parameters $\beta_0, \beta_1, \beta_2, \gamma_1, \gamma_2$ fitted to YtM.

Default starting values of Swenson parameters are in the following table 2.

β0	4,5 %
β1	-4 %
β2	0
β3	0
γ1	2
γ2	1

Table2: Default starting values of Swenson parameters

For more information you can read [5] and [6].

11. Smith-Wilson Model

The aim of this method is to assess the discount factor $P(t)$ for all maturities t, $t > 0$. Input parameters of this method are macroeconomic ultimate long term forward rate (UFR) and alpha (α). UFR is reached asymptotically.The extrapolated forward rates converge faster to UFRfor higher alpha (mean reversion).

Suppose that we have *N* financial instruments as input and *J* is the number of different dates at which a cash payment has to be made. Denote m_i as the market price of instruments *i* at valuation date, for *i* = 1, 2, 3,...*N*. The cash flows c_{i1}, c_{i2}, ...,c_{ij} of instrument *i* at time u_1, u_2, ..., u_J.

$$P(t) = e^{-UFR*t} + \sum_{i=1}^{N} \zeta_i * \left(\sum_{j=1}^{J} c_{i,j} * W(t,u_j) \right), t \geq 0 \qquad (4)$$

Where $W(t,u_j)$ is symmetric Wilson function definedas:

$$W(t,u_j) = e^{-UFR*(t+u_j)} * \left\{ \alpha * \min(t,u_j) - 0{,}5 * e^{-\alpha*\min(t,u_j)} * \left(e^{\alpha*\min(t,u_j)} - e^{-\alpha*\min(t,u_j)} \right) \right\} \qquad (5)$$

ζ_i ,*i* = 1, 2, 3,..., *N*, areunknown parameters needed to fit the actual yield curve.

$$m_i = \sum_{j=1}^{J} c_{i,j} * P(u_j), \; i = 1, 2, ...N$$

In vector space notation we can find the parameters ζ_i as follows:

$$\zeta = \left(\mathbf{CWC^T}\right)^{-1}(\mathbf{m} - \mathbf{C\mu})$$

with:

$\mathbf{C} = c_{i,j}$ i = 1, 2, ...N; j = 1, 2, ...J is the cash flow matrix.

$\mathbf{W} = W(u_i, u_j)$ i = 1, 2, ...J; j = 1, 2, ...J is the matrix of certain Wilson functions.

$\mathbf{m} = (m_1, m_2, ..., m_N)^T$ is the vector of market prices.

$\boldsymbol{\mu} = (e^{-UFR*u_1}, e^{-UFR*u_2}, ...e^{-UFR*u_J})^T$ is the vector of asymptotical long term behavior of discount factor.

Now we can plug these parameters $\zeta_1, \zeta_2, ...\zeta_N$ into the discount function*P*(*t*) (4) and get the discount factors for all maturities. Once we have these discount factors, we can calculate spot rates, forward rates, par rates etc.

Default starting values of Smith-Wilson parameters are in this table 3.

UFR	4,2 %
α	0,1

Table3: Default starting values of Smith-Wilson parameters

For more details you can read the paper [4].

12. LINEAR INTERPOLATION METHODS

Four linear interpolation methods of spot rates, discount factors, par rates and 1-year forward rates can be found in the tool. For linear interpolation methods, following formula is used:

$$R(t) = \frac{t - t_{i-1}}{t_i - t_{i-1}} R(t_i) + \frac{t_i - t}{t_i - t_{i-1}} R(t_{i-1}), \qquad (6)$$

which is defined for$t_{i-1} < t < t_i$, where R_{ti} are discretely compounded zero rates, discount factors, 1-year forward ratesor par rates according to the selected method.

A basic approach to creating yield curves is similar for all linear interpolation methods. Input financial instruments determine cash flow payments at each cash flow payment dates. Output yield curves arecreated from modeled dirty prices (or modeled Yield to Maturity calculated from modeled clean prices). Modeled dirty prices are calculated as the scalar product of cash flows and discount factors. Finding discount factors as a function of entered rates is therefore a crucial part of constructing yield curves. Minimizing the sum of squared differences between these modeled dirty prices and Dirty Market Prices (DMP) providesoptimal values of entered interpolated rates fitted to DMP.Optimal values of entered rates fitted to Yield to Maturity (YtM) are obtained by minimizing sum of squared differences between modeled YtM and market YtM.

13. LINEAR INTERPOLATION OF SPOT RATES

Times between cash flow payment dates and valuation dateare defined from input financial instruments. Oncethe spot rates at each timeof particular cash floware known by using the formula (6), it can be used the procedure of constructing yield curves described in the paragraph 4.4.

Please take into account that the first time parameter (0 years and 0 months, which cannot be changed) indicates the beginning from which the spot rates are linearly interpolated.

For more details you can read the paper [2].

14. LINEAR INTERPOLATION OF DISCOUNT FACTORS

Findingof discount factor between two time parameters is equivalent to linear interpolation of spot rates method and thus similar formula to (6) is used, where$R(t)$represents the entered discount factor.

Once discount factors at each time of particular cash flow related to input financial instrument are known, it can be used the procedure of creating output yield curves, which was described in the paragraph 4.4.

Please take into account that the first time parameter (0 years and 0 months, which cannot be changed) signifies the beginning from which the discount factors are linearly interpolated.

For more details you can read the paper [2].

15. LINEAR INTERPOLATION OF PAR RATES

Assume that we have n-year Interest rate swap (IRS) with fixed par rate and n-year coupon bond with coupon rate equal topar rate at the par value. Thus Market value of coupon bond is 1 and its Yield to Maturity is equal to par rate. The possible terms of IRS are integer years 1, 2, …50 in the tool. For each year up to last entered time parameter discount factor at particular time is calculated by bootstrapping as:

$$P(t) = \frac{1 - par\_rate_t * \sum_{t=1}^{t-1} \alpha_i * P(i)}{1 + \alpha_t * par\_rate_t}, \qquad (7)$$

where α_i is accrual factorwhich reflects time convention.

It is necessary to know discount factors up to year t-1 for using the formula (7). Thus par rates at the not entered integer years (up to t-1) are linearly interpolated by using the formula (6).

Linear interpolation (formula (6)) of discount factors calculated from the formula (7)is used to determine discount factors at each time of particular cash flow related to input financial instrument. Once the discount factors are known the procedure described in the paragraph 4.4 can be applied for constructing yield curves.

16. LINEAR INTERPOLATION OF 1-YEAR FORWARD RATES

In this method 1-year forward rates are the input parameters. In addition it has to be filled in parameter in 0 years and 0 months(overnight spot rate and also 1-year spot rate from 0 year and 0 months (i.e. 1-year forward rate from 0 to 1). These two necessary parameters are used for calculation of the spot rate for cash flows that lie between 0 year and 1 year (i.e. the first element of formula (8)).If this spot rate is known, zero rate (and then discount factor) foreach time can be calculated by using the following formula (discrete compounding).

$$(1+R_t)^t * (1+F_{t,1}) * (1+F_{t+1,1}) \ldots * (1+F_{t+n,1}) = (1+R_{t+n})^{t+n}, \quad (8)$$

whereR_t is the spot rate between 0 and 1 year and $F_{t+n,1}$ is 1-year forward rate from time *t+n*.

At the first step the spot rate between 0 and 1 year was found and subsequently 1-year forward rates in the formula (8) was calculated based on formula (6).

Once thespot rates at each time(and then discount factors) of particular cash flow related to input financial instrument are known, the procedure of constructing yield curves in theparagraph 4.4 can be used and then optional estimators of entered 1-year forward rates are obtained.

17. CUBIC SPLINE INTERPOLATION

This method is also called piecewise polynomial interpolation. It is used third degree polynomialfunction, which determines a discount factor function. Time interval of interpolated part is dividedinto *n* knot points t_i, where $i = 1, 2, \ldots n$ (in the tool the maximum number of entered knot points is 6). In each *i*-th part of time interval ($1 <= i <= n-1$) the polynomial formula is following:

$$DF(t) = a_i + b_i * t + c_i * t^2 + d_i * t^3, t \leq t_{i+1} \quad (9)$$

Three additional conditions must be met:

- The interpolated values at each end of the interval must be equal to input values (i.e. discount factors in particular time).
- Derivative continuity: the first and second derivatives are equal to the left and to the right in each knot point for$i = 1, 2, \ldots n - 1$.
- *DF*(0) = 1

Note: You can see cubic polynomial in the following form:

$$DF(t) = a_i + b_i(t - t_i) + c_i * (t - t_i)^2 + d_i * (t - t_i)^3, t_i \leq t \leq t_{i+1}$$

It has no effect to fit, possible changes can be found in a_i, b_i, c_i, d_i estimates.

Once discount factors at each time of particular cash flow related to input financial instrument are known, it can be used the procedure of creating output yield curves, which was described in paragraph 4.4.

For more details you can read the paper [2].

18. TIME CONVENTIONS

In linear interpolation methods and cubic spline interpolation it is necessary to find time period between valuation date and date of cash flow payment of the financial instrument. Modified Following Business Day Convention is used for this purpose. Modified Following Business Day Convention is aprocedure used for adjusting payment dates that respects days that are not TARGET Business Days.

TARGET Business Day is a day (other than Saturday or Sunday) on which the Clearing System and the Trans-European Automated Real-Time Gross Settlement Express Transfer System (TARGET) are open.

If payment day falls on Saturday or Sunday then it is rolled forward to the next TARGET Business Day, unless that day falls in the next calendar month, in which case the payment day rolls backward to the immediately preceding TARGET Business Day.

This procedure uses traditional day count conventions (30US/360, Act/Act, Act/360, Act/365, 30E/360), but the date of cash payment is changed as mentioned above.

For more details about computing traditional day count conventions you can read the paper [2].

19. LITERATURE

[1] CIPRA, Tomas. *Finanční a pojistné vzorce*. Praha: Grada, 2006. ISBN 80-247-1633-X.

[2] HAGAN, Patrick S. a WEST, Graeme. METHODS FOR CONSTRUCTING A YIELD CURVE. In: *Financial Modelling Agency* [online]. [cit. 2012-09-21]. Availablefrom: http://www.finmod.co.za/interpreview.pdf

[3] NELSON, Charles R. a SIEGEL, Andrew F. Parsimious Modeling ofYieldCurves. In: *Department ofMathematicalSciences* [online]. [cit. 2012-09-21]. Availablefrom: http://www.math.ku.dk/~rolf/teaching/NelsonSiegel.pdf

[4] NELSON, Charles R. a SIEGEL, Andrew F. Technicalnote on the Smith-Wilson Method*In: TheFinancialSupervisoryAuthorityofNorway* [online]. [cit. 2012-09-21]. Availablefrom: http://www.finanstilsynet.no/Global/Forsikring%20og%20pensjon/Skadeforsikring/Tilsyn%20og%20overv%C3%A5king/Rapportering/A_Technical_Note_on_the_Smith-Wilson_Method_100701.pdf

[5] THOMAS, Michael and Eben MARÉ. Long Term Forecasting and Hedging. In: *Actuarial Society* [online]. [cit. 2012-09-21]. Availablefrom:

http://www.actuarialsociety.org.za/Portals/1/Documents/6c1c9e5b-0df8-4fa6-a40f-8afac0443be6.pdf

[6] SEPPÄLÄ, Juha a VIERTIÖ, Petri. The Term StructureofInterestRates: Estimation and Interpretation. In: *Econometricsatthe University of Illinois* [online]. [cit. 2012-09-25]. Availablefrom: www.econ.uiuc.edu/~seppala/econ563/tse.pdfb