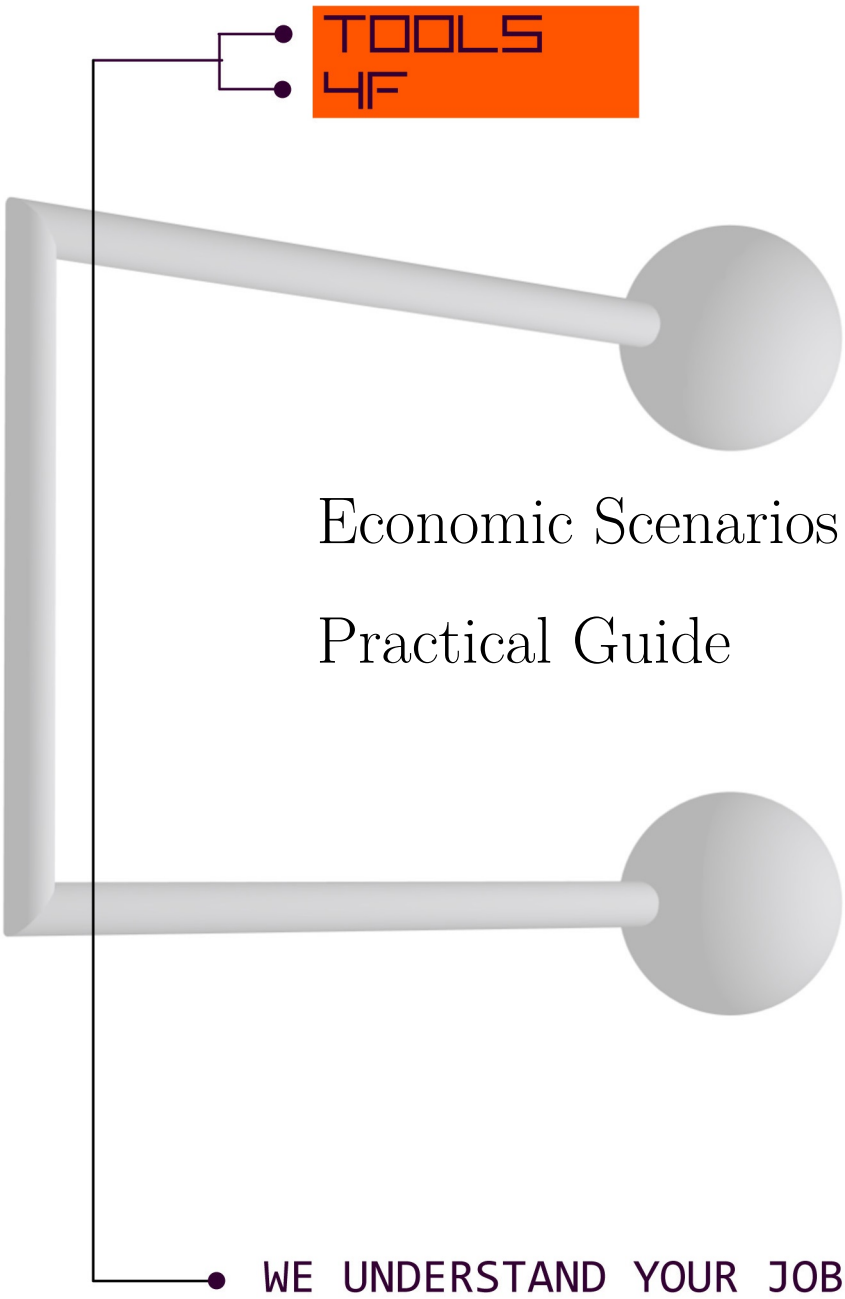

Economic Scenarios As a Service

Practical Guide

1 Economic Scenarios As a Service

Economic Scenarios As a Service (ESAS) is a service providing risk-neutral economic scenarios that are used for valuation of embedded options, guarantees and other financial derivatives. A Scenario in this context is one possible future development of the market variables. The ESAS provides scenarios for[1]

- Discount Factors,
- Discount Rates,
- Zero-Bond Prices,
- Spot Zero Rates,
- PAR Rates,
- Forward Zero Rates,
- Forward Swap Rates,
- Equity Prices,
- Foreign-Exchange (FX) Rates.

For technical details, please see the ESAS Technical Documentation.

The underlying model satisfies recommendations published by Society of Actuaries (link).

[1] Bond prices and interest rates are risk-free.

2 How the ESAS Works?

The ESAS provides a set of scenarios on a regular basis for valuation purposes, together with all necessary documentation satisfying professional and regulatory requirements.

The following documents are supplied.

- **Scenarios Data.** Files with figures in a pre-agreed structure. Usually, .csv files with a matrix structure, where individual files represent financial market values, columns represent time, and rows represent individual scenarios.
- **Calibration and Validation Report.** Documentation of all practical steps that were taken in order to generate a particular set of scenarios and validation tests associated with one valuation date.
- **ESAS Technical Documentation.** A detailed general description of mathematical models, methods, and validation tests that were used to generate Scenarios Data, and Calibration and Validation Report.

3 How to Use a Set of Scenarios for Valuation

The risk-neutral scenarios generated by the ESAS are for valuation purposes, and their usage is straightforward. According to the no-arbitrage theory employed to build the model every asset can be valued using a so-called risk-neutral pricing formula

$$Price = \mathbb{E}\left[Payoff * DiscountFactor\right].$$

The pricing formula tells us that any derivative or guarantee can by valued by taking the expectation of discounted payoff of financial derivative or guarantee. However, instead of evaluating complicated theoretical expectation, the following can be done:

1. calculate discounted payoff for every particular scenario,
2. compute an average discounted payoff across all scenarios, which will be the required price.

If the number of scenarios is high enough[2], the average discounted payoff will be very close to the expected discounted payoffs from the pricing formula.

4 Calibration and Validation Report

The Calibration and Validation Report is a document that provides justification that the set of scenarios is appropriate for valuation purposes.

The calibration part covers practical steps that were taken to generate a particular set of scenarios associated with one valuation date. This includes a description

[2]The more complex the valued derivative or guarantee is, the more scenarios are needed for a close approximation. Usually, 1000 or more scenarios are used.

of input market data, parameters of the underlying mathematical model, and possibly description and justification of any deviation from the general methodology described in the ESAS Technical Documentation.

The Validation should, according to SOA ESG Practical Guide, include market consistency test and so-called martingale or 1=1 test. Additionally, the report includes some other tests showing the appropriateness of supplied scenarios. In particular, the report includes the following tests.

- **Market Consistency Test.** An evidence that the used model provides a good approximation to the market, usually performed by comparing model prices with market prices (e.g., option or swaption prices).

- **Martingale Test (1=1).** The purpose of this test is to demonstrate that the set of scenarios does not allow arbitrage opportunities. The non-existence of arbitrage opportunities is a key assumption of pricing models, for more details, see chapter 6 on Risk-Neutrality. According to the pricing theory, any asset in risk-neutral scenarios should, on average, generate a return equal to the risk-free rate. This is implemented using investment one unit of currency into a zero-coupon bond, stock, or foreign currency.

- **Convergence Test.** The average discounted payoff is an approximation to the expectation of discounted payoff from the pricing formula. The higher number of scenarios is, the closer the average will be to the theoretical expectation on average. The convergence test provides evidence that a particular set of scenarios has achieved the convergence for selected derivatives. The test is based on a comparison of exact model prices of financial derivatives given by the pricing formula with simulated prices of the same derivatives. Since we need to evaluate the pricing formula to obtain the model prices, the implementation is based on less complicated derivatives for which the

evaluation of the pricing formula is possible, typically plain vanilla options and swaptions.

5 Input Parameters of Scenarios

In order to generate a set of scenarios, input parameters are needed. The input parameters may be provided by the client, or upon a consultation, they may be provided by ESAS according to client needs. The input parameters can be divided into four basic categories.

1. **Domestic Yield Curve.** Specifying domestic yield curve; e.g., EUR Market Yield Curve, EUR EIOPA Yield Curve, etc.

2. **List of Market Variables on Ouput.** A list of market variables that are requested as output accompanied by additional parameters associated with a given variable. For example, spot zero interest rate requires as an additional input time to maturity (in years). For complete information on parameters related to Market Variables, please, see the table below.

3. **Simulation Parameters.** This category includes only two parameters,

   - a horizon of the scenarios (e.g., scenarios for the next 20 years),
   - a number of scenarios (usually at least 1000).

4. **Formatting Parameters.** Since the scenarios are further used as input data to the internal liability model, it may be convenient to format the scenarios, so it is compatible with the internal liability model of an individual client. This mainly includes date and time formatting; frequency of data[3]

[3]Do not mistake the frequency of data with a frequency of supplies. For example, daily data may be supplied on a monthly basis. In other words, market variables simulated day by day are provided every month.

(e.g., daily, monthly, annually, custom dates); special symbols used for separating values, decimal marks, and new lines; and so on. These parameters are set according to individual consultation and tested with sample files.

The full list of market variables available with required parameters is in the table 1.

Market Variable	Parameters
Discount Factor	Discount One Period Backward or to Valuation Date
Discount Rate	Discount One Period Backward or to Valuation Date
Zero-Bond Price	Time to Maturity
Spot Zero Rate	Time to Maturity
PAR Rate	Time to Maturity Frequency of Coupon Payments
Forward Zero Rate	Time to Forward Maturity Time to Bond Maturity
Forward Swap Rate	Time to Forward Maturity Swap Tenor (lifetime) Frequency of Swap Payments
Equity Prices	
FX Rate	Domestic or Foreign Denominator

Table 1: List of available market variables. All interest rates assume continuous compounding.

6 What Risk-Neutral Means?

The ESAS is built on an arbitrage-free pricing theory that assumes there are no arbitrage opportunities in the market. This assumption holds well for liquid

markets in normal times and provides mathematical tools allowing us to uniquely determine prices of assets and liabilities. However, to mathematically achieve an environment without arbitrage opportunities, we construct an artificial probability measure that is different from a real-world probability of having some scenario; and we enforce every asset to generate a risk-free rate under this artificial measure. Consequently, the risk-neutral scenarios provided by ESAS are not real cash-flow projections, and cannot be used so. **The risk-neutral scenarios can be used for valuation purposes only.**

7 ESAS Limitations

The ESAS can currently provide scenarios for all variables stated in chapter 1. However, there are some limitations to Equity and FX scenarios.

1. **Option Market Existence.** Since the risk-neutral probability is different from the real-world probability under which we observe historical data, the model cannot be calibrated to historical data. Therefore, it is typically calibrated to traded financial derivatives such as plain vanilla European options. Consequently, it is possible to calibrate the model only for stocks and exchange rates with existing option markets.

2. **Foreign Interest Rates** Currently, the ESAS assumes that foreign interest rates are deterministic; therefore, it is not possible to generate scenarios for foreign interest rates.

8 Definitions and Notions

- **Valuation date.** The underlying ESAS model is calibrated to market data observed at particular date that is called Valuation date.

- **Time to Maturity.** The term Time to Maturity or Term to Maturity is time till the end of some contract, typically bonds, options, forwards, and so on.

- **Time to Forward Maturity. (Forward Rate)** Forward is a contract between two parties to buy or sell at a specified future time (Forward Maturity) for a pre-agreed price. Forward Zero Rate can be seen as a yield that would generate a bond with a pre-agreed price, and Forward Maturity is the time when the parties have an obligation to buy/sell the bond. (Typically, the position is cleared without buying/selling an actual bond.) Time to Forward Maturity of Forward Swap Rate is then a date when parties have an obligation to enter an interest rate swap.

- **Swap Tenor.** The interest rate swap is a contract between two parties to exchange interest rate payments - floating rate and fixed rate defined in the contract. The interest rate payments are exchanged over the time period that is commonly denoted as a tenor.

- **Frequency of Swap Payments.** The swap payments are exchanged on a regular basis. The frequency affects the value of the swap, and consequently, the market value of Forward Swap Rate. The frequency is expressed in years, i.e., 1/2 means that interest rate payments are exchanged every 6 months.

- **Frequency of supplies.** The Scenarios Data may be supplied on a regular basis - monthly, quarterly, annually, etc. For example, an insurance company is interested in values of embedded options at the end month of each month. Therefore, they receive a new set of scenarios at the end of each month.

- **Frequency of data.** The frequency of Data determines time points at which the market variables are observed within a set of scenarios. The scenarios

are, by default, simulated day by day, which means for each scenario value of a particular market variable today, tomorrow, day after tomorrow, and so on is known. However, the typical financial derivatives and embedded options do not have payoff functions dependent on everyday market values. Therefore, the Scenarios Data includes only selected days, e.g., the monthly frequency with a value corresponding to the end of the month.

- **Zero-Bond Price** Price of zero coupon bond with nominal (face) value 1.
- **Spot Zero Rate** Interest rate, or rather yield, that is associated with zero coupon bond with a given maturity.
- **PAR Rate** PAR Rate (or Yield) is a coupon rate such that price of coupon bond is equal to the nominal (face) value of the bond.
- **Forward Zero Rate** Interest rate paid from Forward Maturity till a pre-agreed future date denoted as Bond Maturity.
- **Forward Swap Rate** A fixed interest rate of interest rate swap that starts in the future (Forward Maturity), pays interest over its lifetime (Swap Tenor) with pre-agreed frequency of payments.
- **FX Rate** Foreign exchange rate between domestic and foreign currency. When denominator is domestic (foreign), then the rate is in form foreign/domestic (domestic/foreign).