TOOLS 4F

ACTUARIAL FUNCTIONS 1_05

User Guide

for MS Office 2007 or later

CONTENT

1 INTRODUCTION

The application called Actuarial function 1.0 is specifically designed to help actuaries in their life calculations created in MS Excel environment through a library of basic actuarial functions in the form of add-in to Microsoft Excel.

The purpose of Actuarial function 1.0 is to cover basic actuarial functions (see chapter 6) and to support the user in a highly qualified work with validated and automated calculations without any additional time demands.

The potential benefits include:

- Enhanced productivity
- Decreased time to final result
- Standardized and optimized work

Actuarial function 1.0 is a small and lightweight application with minimal memory consumption.

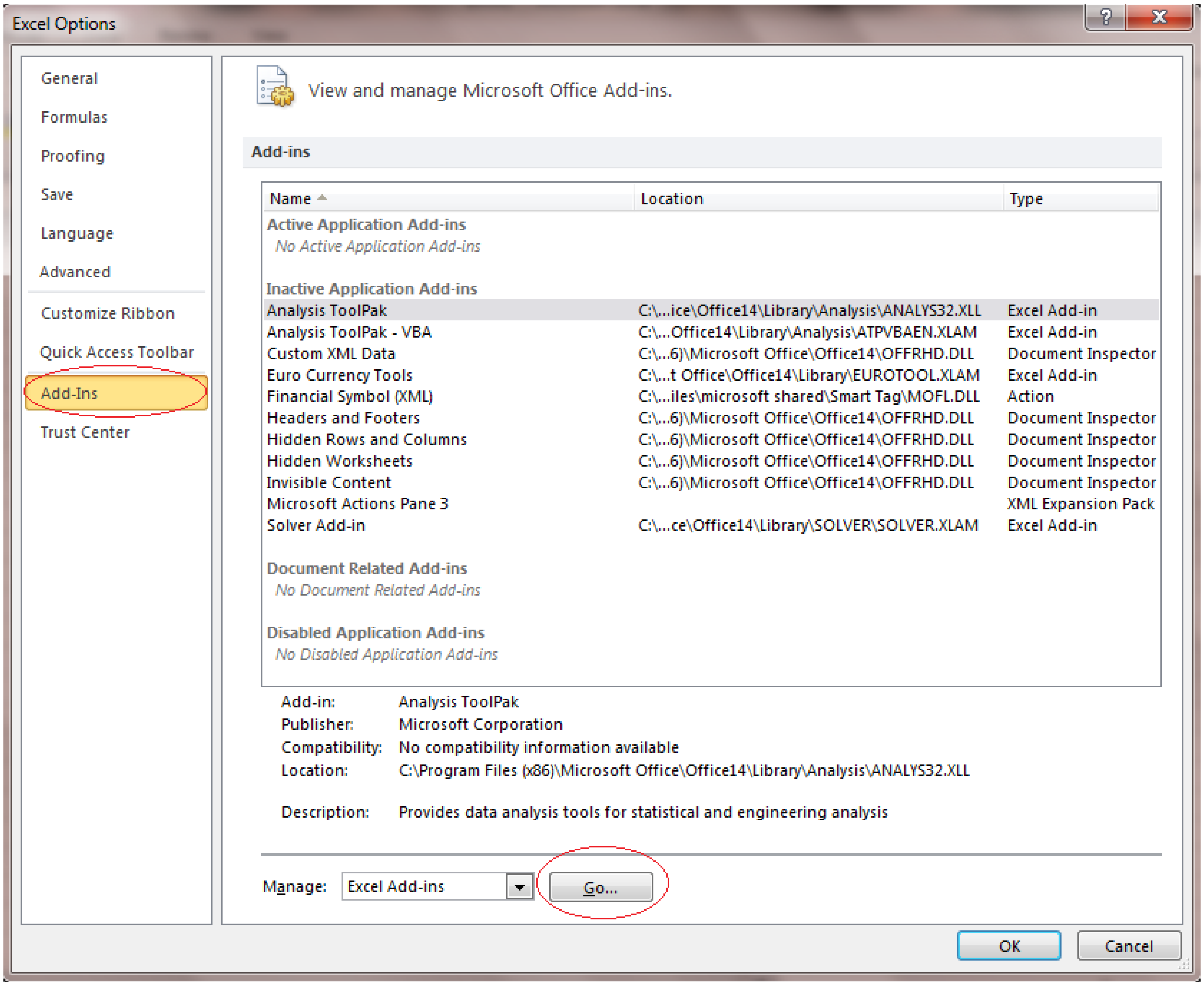
This user´s guide will cover the actuarial function tasks associated with Microsoft Excel. It is assumed that the reader is familiar with the Microsoft Excel environment.

2 INSTALLATION PROCEDURE

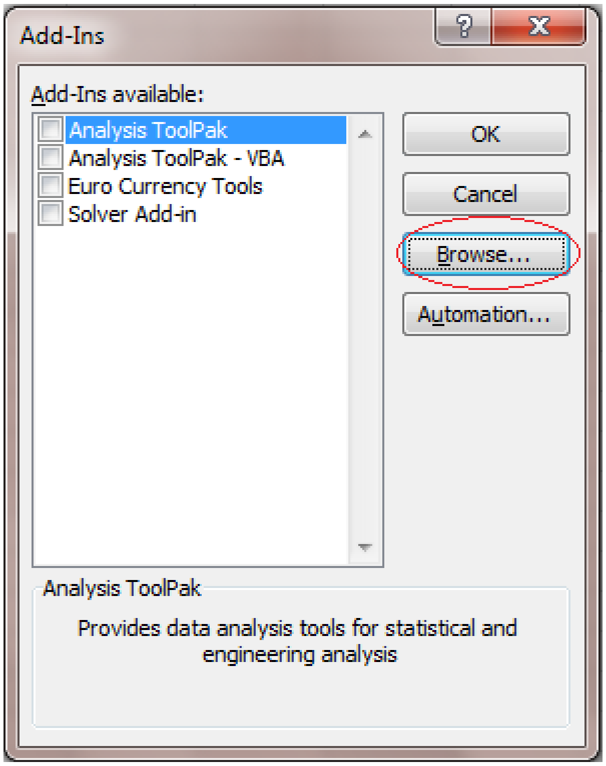
To run the application, you need to download the .xla file and save it on your disc.

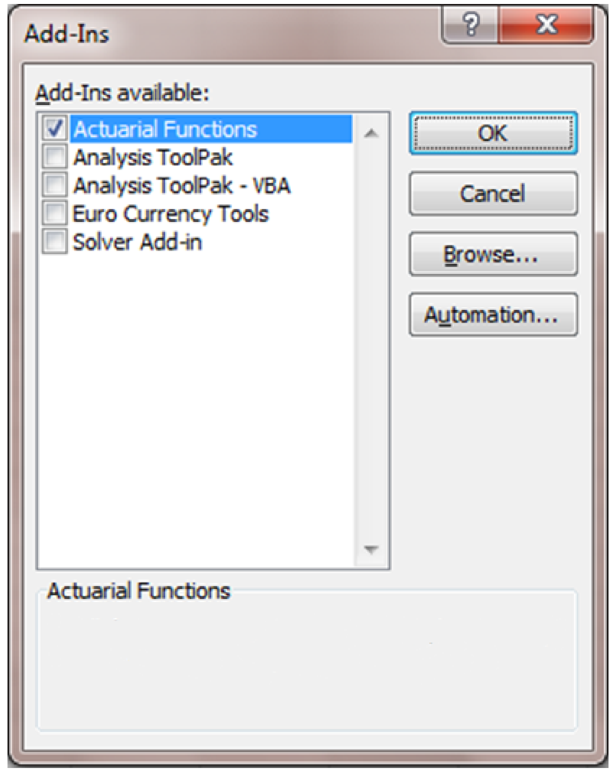
NOTE: .xla files do not work without opening through MS Excel. It is necessary to open the MS Excel first and through the *File –> Open* find the .xla file.

If you want to add Actuarial function to MS Excel and have the function available whenever your MS Excel is opened, click on *File -> Option* and choose Add-Ins. Click on the “Go…” button at the bottom of the form shown below.



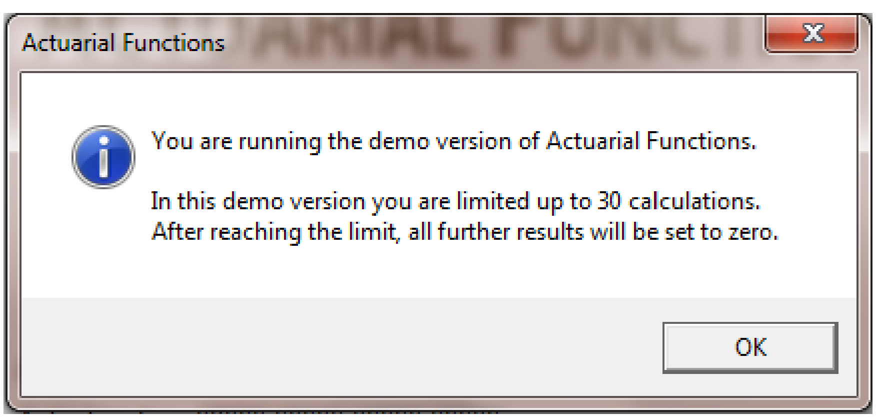
The window with a list of available Add-Ins appears. Click on the "Browse…" button and select the .xla file on your disc and click on "OK". Then the Actuarial function is available in the list of the Add-Ins. Check the box and click on "OK". The actuarial function is available in the "User defined function" (for more information see chapter 4).



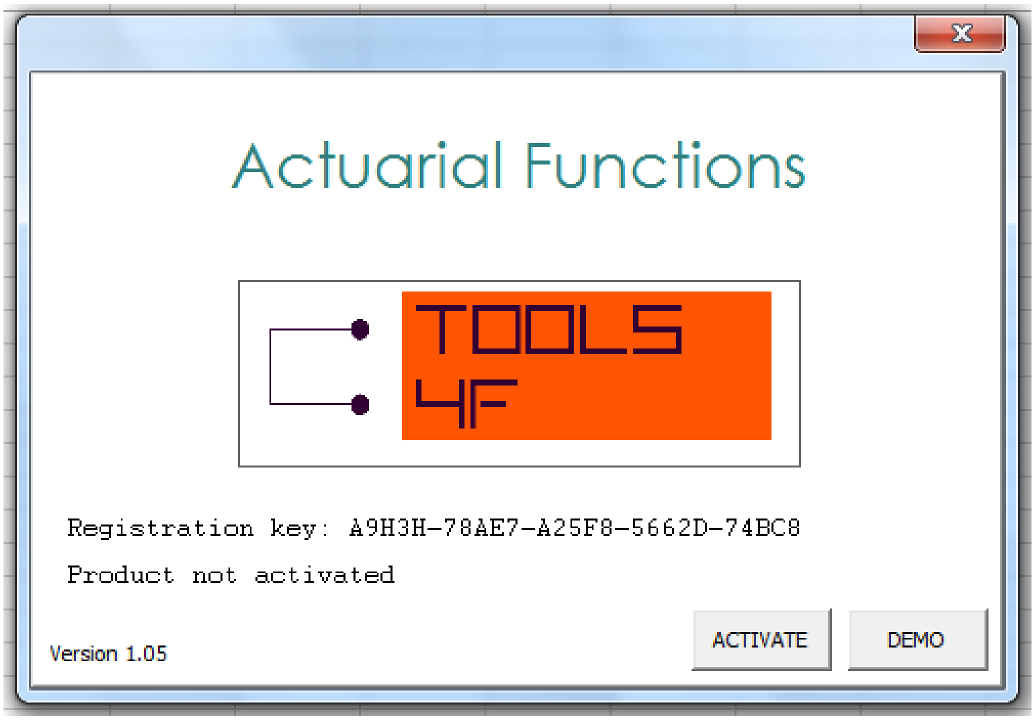


3 Demo Version and Activation

After opening the file, you will be informed about the demo version running. In the demo version, you are limited to the maximum of 30 calculations. Once you reach the limit, all further results will be set to zero. Click "OK" to continue.



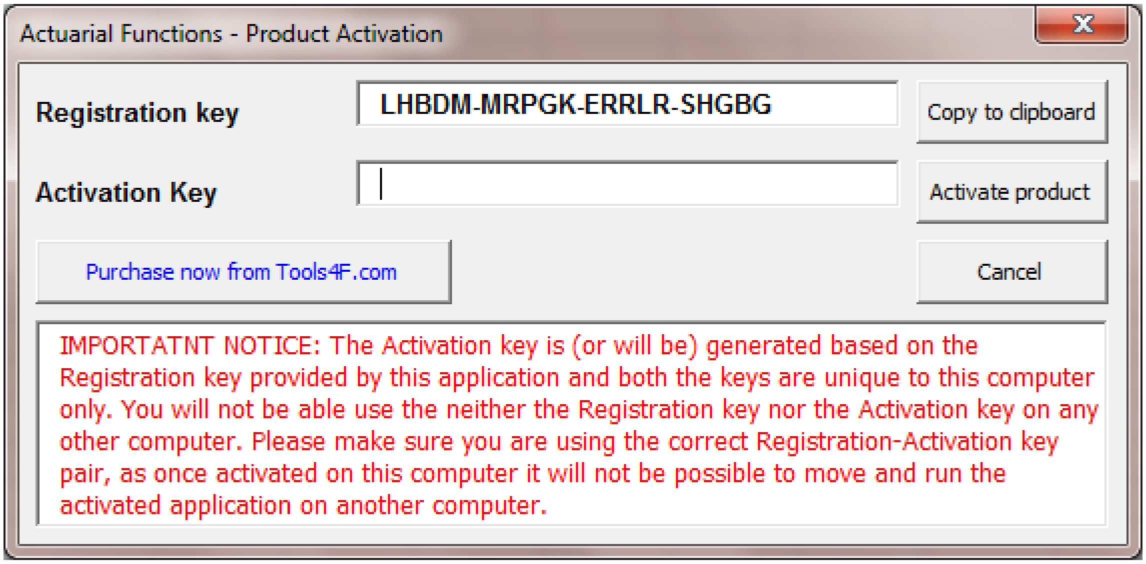
Now, the Welcome screen appears and you have the option to activate the full version of the Actuarial function. If you want to run the demo, click on the “Demo” button. Once you want to activate the full version, click on “Activate”.



As you already have the registration key, enter it in the “Activation key” cell and click on the “Activate product” button.

In case you do not have the Activation key, click on the “Purchase now from Tools4F.com” button and insert the Registration key (the “Copy to clipboard” button makes the process smoother) in the purchase form. Once the license is ordered and paid, the Activation key is sent to you and you can activate the full version.

NOTE: The Activation key is generated by means of the Registration key provided by this application and both of the keys are unique to one computer only. You will not be able to use neither the Registration key nor the Activation key on any other computer. Please make sure you are using the correct Registration-Activation key pair, as, once activated on one computer, it will not be possible to move and run the activated application on another computer.



4 USING FORMULAS AND SYNTAX

Actuarial functions 1.0 includes a set of mathematical predefined formulas which are used to simplify the process of inserting functions into the worksheets and eliminate errors.

NOTE: Once you work with Actuarial function, all the data you use must be in one workbook. It is not possible to use the data which are in other workbooks than those with the calculations.

Before you start working with the function, it is necessary to set/define the input data ***lx***-number of the living according to their age.
The data table must have the following structure:

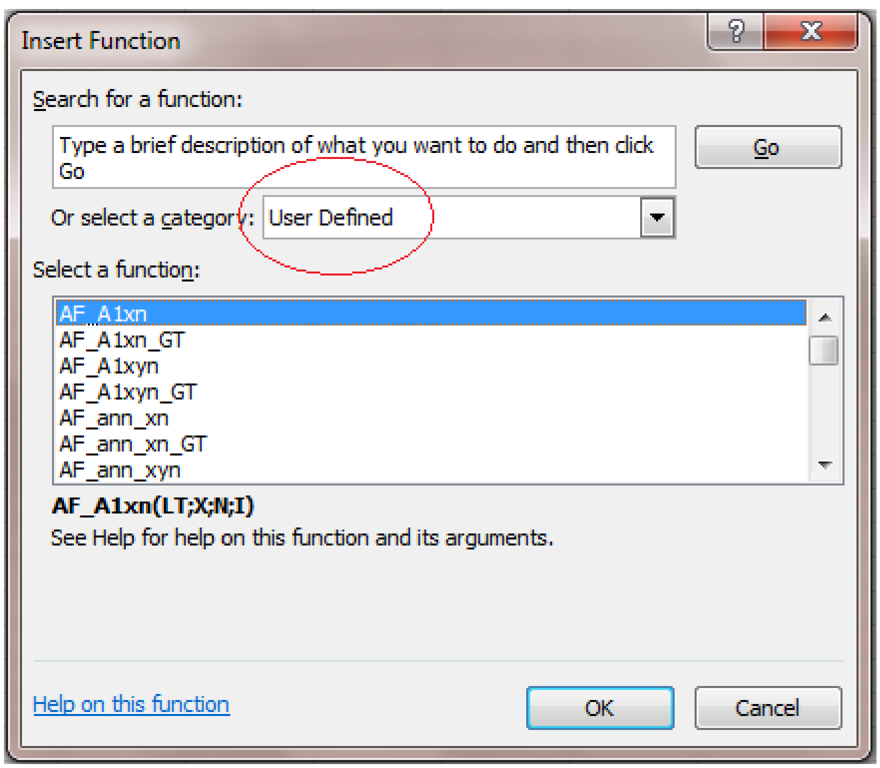
- index age (integer) in the first column (column A) and
- value lx (real)in the second column (column B) as shown in the following picture.

A	B
Age	lx
0	100000
1	99571
2	99531
3	99509
4	99485
5	99469
6	99455
7	99439
8	99422
9	99403

This table has to be placed on the named worksheet, the name of which is *"LT_M"* in our example as shown in the picture below.

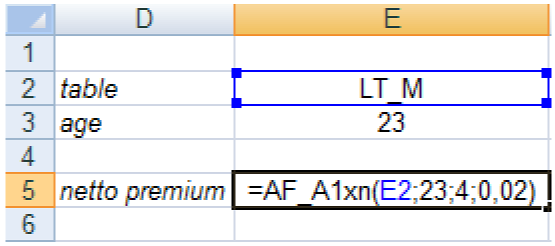
All the Formulas in this Actuarial Function add-in start with the prefix "AF" and all their arguments can be specified by value or by reference.

To display the functions click on the cell where you want to apply the function, click the Insert Function button (*fx*) to the left of the formula bar and click on the function category "User defined". Once you have selected a function, click OK.



Once you have chosen the function, Excel will display a syntax window to help you with constructing the function. For further explanation of each argument see chapter 5.

Let us calculate a simple example here. The function *AF_A1xn(LT; X; N; I)* (single netto premium for pure death cover (term)) has four arguments. The first one is called *LT*. As described in chapter 5, *LT* stands for "Life Table". It is the name (string) of the sheet with the lx table, in our case it is *LT_M*. As it is a string variable, it has to be set within the quotation marks. If the name of the worksheet with the lx table is *"LT_M"* and is placed in cell E2, you can use syntax *=AF_A1xn(E2;23;4;0,02).*

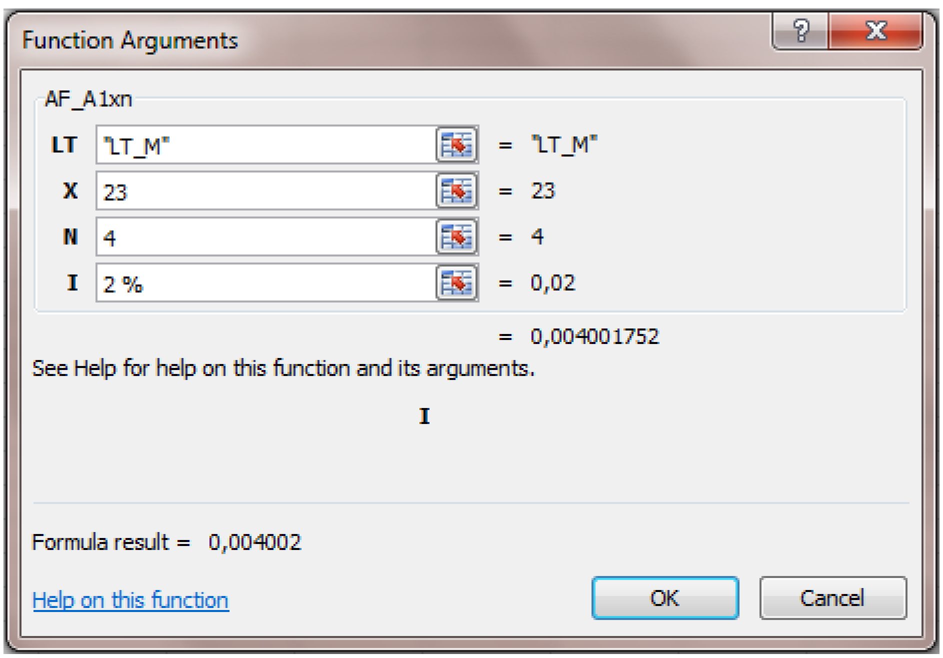


The other argument is *X* - Age which is the entry age of the insured person in years (integer).

The third argument is *N* – Duration. Duration is a period in number of years (integer) (typically the policy period).

The fourth argument is *I* – Interest. Interest is the annual interest rate (real) used to discount the payments. There are two possibilities how to write this argument. One option is to write a decimal number (e.g. 0.02), the other one is to write percentage (2 %).

When this is finished, click OK in the syntax window to insert the function into your worksheet.



Of course, even here, when you use cell references in the Excel formulas, the formulas will automatically update whenever the relevant data in the spreadsheet change.

Other examples of syntax

Function *AF_ann_xn(LT; X; N; PF; I; PT)* (present value of life annuity for a limited number of years) has six arguments.

The arguments *LT* – Life table, *X* – age, *N* – duration and *I* – Interest are explained above.

PF – Payment frequency is a number of payments within one year (1 - annually, 2 – semi-annually, 4 - quarterly, 12 - monthly). As it is a number, there is no need to put it in the quotation marks.

PT – Payment type takes values 1 - for payments at the beginning of the period or 0 - for payments at the end of the period.

=AF_ann_xn("LT_F";30;10;2;0,04;0)

Let us calculate function *AF_I_A1xn(LT; X; N; I; INF; IT; IR)* (single netto premium for pure death cover (term) with increasing sum assured), which has seven arguments.

The arguments *LT* – Life table, *X* – age, *N* – duration and *I* – Interest are explained above.

INF – Increase frequency is an increase frequency during one year (integer). This argument accepts the values 1 - annually, 2 - semi-annually, 4 – quarterly or 12 - monthly.

IT– Increase type is the type of payment increasing. Possible values can be 1 for geometric or 2 for linear/arithmetic.

IR–Increase rate is the annual nominal increase rate (real) used for payment increase. For INF = 1, IR can be 100 % or less. If the increase frequency (INF) selected is higher than annual (=1), the increase related to the sub-annual period is IR*freq, e.g. for monthly frequency (=12) the monthly increase is IR*12.

First we consider INF = 1, IR = 100 %.

=AF_I_A1xn("LT_M";25;5;0,03;1;2;100%) = 0,013709247

Now we consider the same interest rate, but increase frequency during one year is semi-annually. That means INF = 2, IR = 200 %.

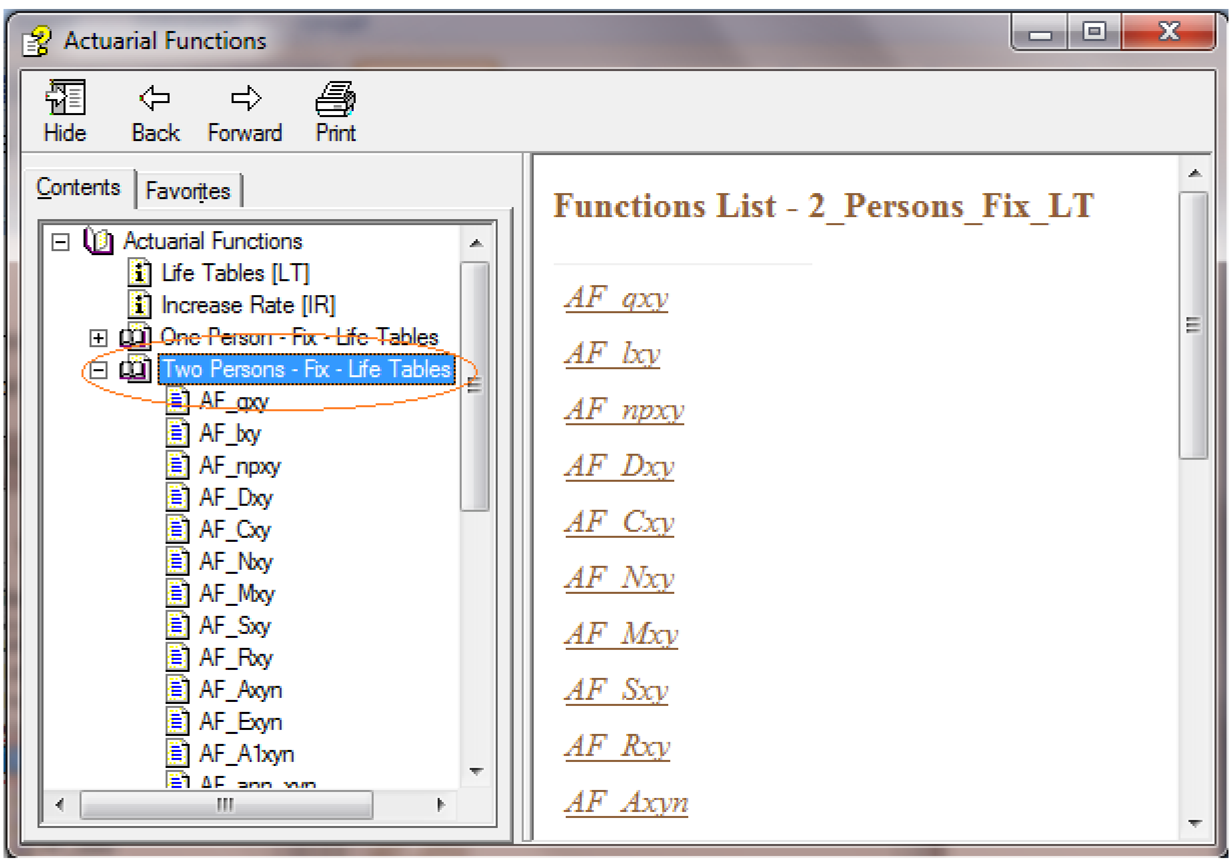
***=AF_I_A1xn("LT_M";25;5;0,03;2;2;200%)* = 0,025277652**

Other possibilities

Besides working with the Life Table, you can also work with Generation Life Tables (*GT*). *GT* must have the following structure: birth year (integer) in the first column (column B), age (integer) in the second column (column C) and lx (real) in the third column (column D)

B	C	D
Year	Age	lx
1920	0	100 000
1920	1	99 571
1920	2	99 531
1920	3	99 509
...	...	...
1921	0	100 000
1921	1	99 575
1921	2	99 536
1921	3	99 514

This Actuarial functions application also includes the calculation for 2 persons. All functions can be seen in the Formula bar or in Help as shown in the picture bellow.



Let us calculate a simple example. The function *AF_A1xyn(LT_X; LT_Y; X; Y; N; I)*(Single netto premium for pure death cover (term) for 2 lives) has six arguments.

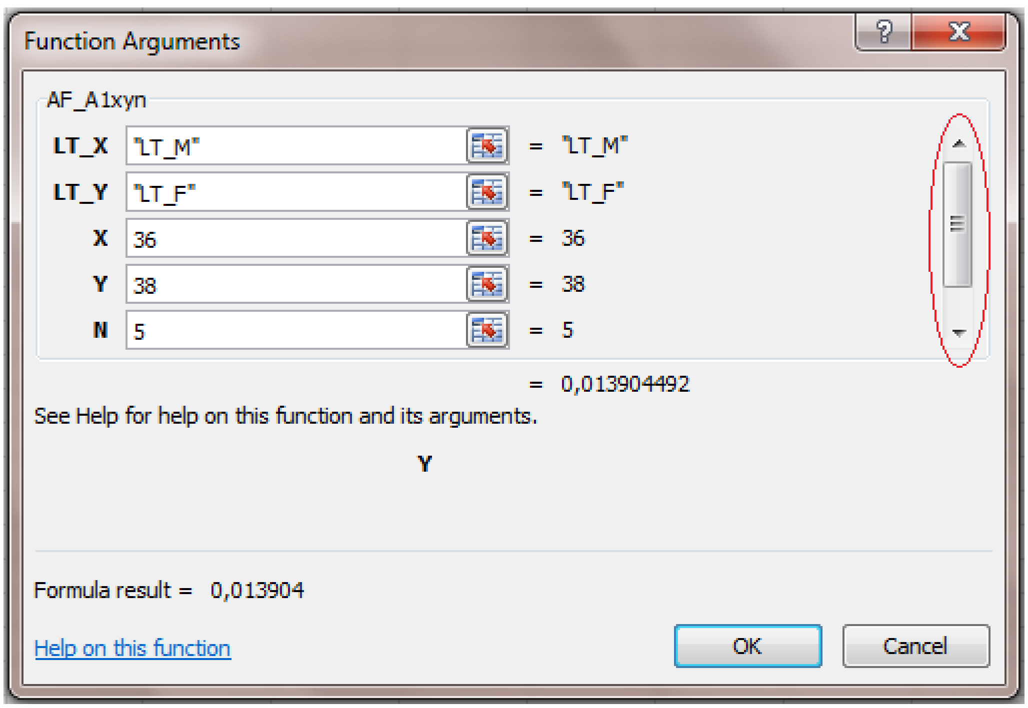
The first two arguments are Life Tables for the persons. In this example, we consider man *(LT_M)* and woman *(LT_F).*

X is the age of the first person, Y is the age of the second person.

N is the Duration. Duration is a period in number of years (integer) of the calculation (typically the policy period, etc.).

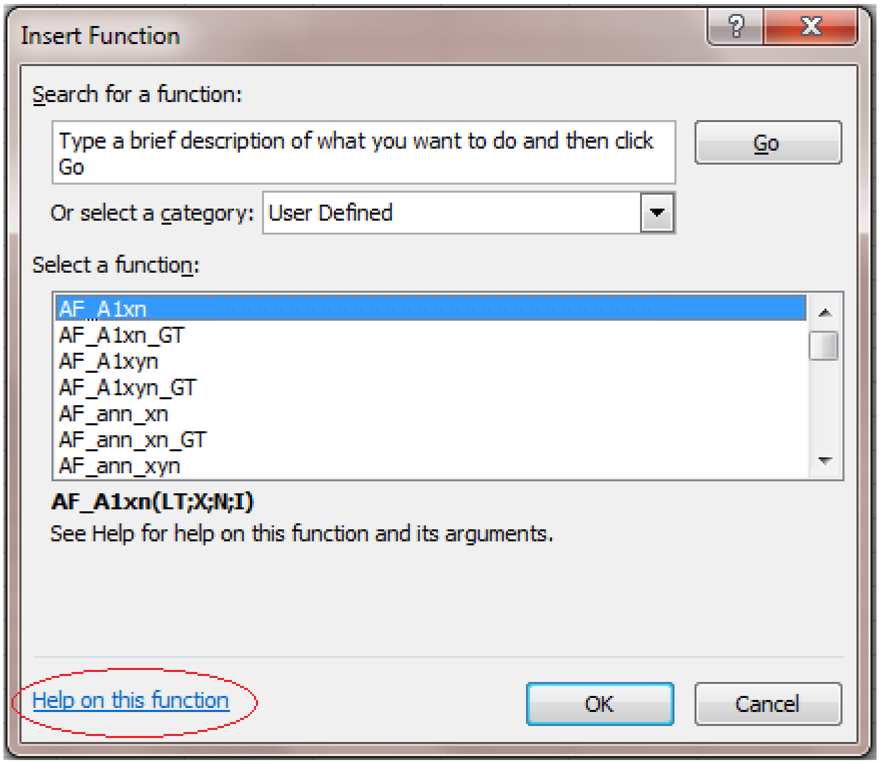
You need to roll down to see the last argument. *I*– Interest is the annual interest rate (real) used to discount the payments. The decimal number or percentage can be input.

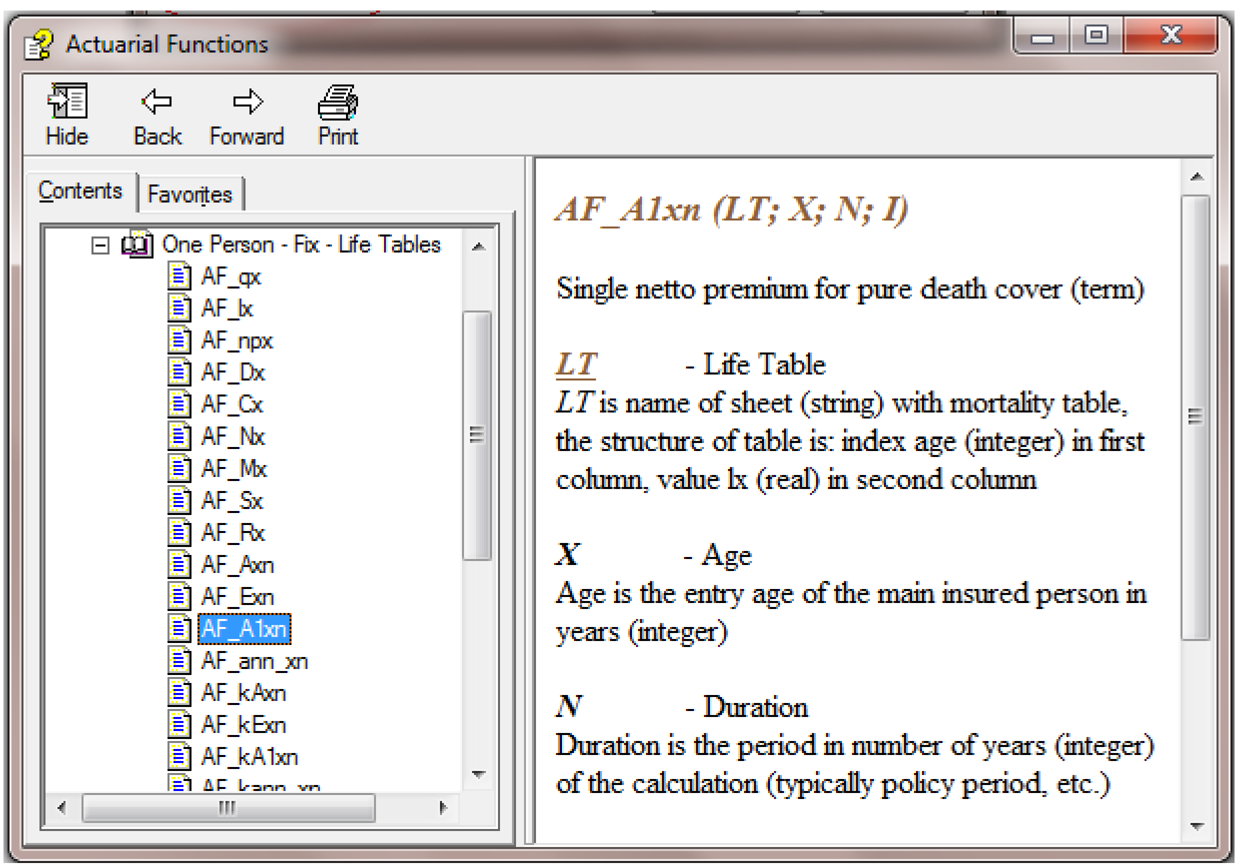
When this is finished, click OK in the syntax window to insert the function into your worksheet.



5 USING THE HELP

If you need extra help in creating your function, click on the “Help on this function” reference in the bottom left corner of the Insert function dialog box. The Help will then be displayed (as in the following picture).


Insert Function
Search for a function:
Type a brief description of what you want to do and then click Go
Go
Or select a category: User Defined
Select a function:
AF_A1xn
AF_A1xn_GT
AF_A1xyn
AF_A1xyn_GT
AF_ann_xn
AF_ann_xn_GT
AF_ann_xyn
AF_A1xn(LT;X;N;I)
See Help for help on this function and its arguments.
Help on this function
OK
Cancel


Actuarial Functions
Hide
Back
Forward
Print
Contents
Favorites
One Person - Fix - Life Tables
AF_qx
AF_lx
AF_npx
AF_Dx
AF_Cx
AF_Nx
AF_Mx
AF_Sx
AF_Rx
AF_Axn
AF_Exn
AF_A1xn
AF_ann_xn
AF_kAxn
AF_kExn
AF_kA1xn
AF_A1xn (LT; X; N; I)
Single netto premium for pure death cover (term)
LT - Life Table
LT is name of sheet (string) with mortality table, the structure of table is: index age (integer) in first column, value lx (real) in second column
X - Age
Age is the entry age of the main insured person in years (integer)
N - Duration
Duration is the period in number of years (integer) of the calculation (typically policy period, etc.)

On the left side, the list of functions is displayed. To learn more about other functions, click on the name of the required function and the Help will be displayed on the right side accordingly.

The Help can be opened directly from the folder where it is saved.

NOTE: The file *.chm and *.xla must be saved in the same folder to work with the Help properly.

If you have any trouble in finding the Help for a particular topic or the Help provided is not sufficient, send an email to us (info@tools4f.com) and our support team will contact you. Your feedback helps us to improve our documentation, so we welcome your input.

6 NOTATION

LT	Life Table	LT is the name of the sheet (string) with mortality table, the structure of the table is: index *age* (integer) in the first column, value *lx* (real) in the second column.
X	Age	Age is the entry age of the main insured person in years (integer)
N	Duration	Duration is the period in number of years (integer) of the calculation (typically the policy period, etc.)
K	Deferred period	Deferred period is the number of years (integer) of deferred payments
PF	Payment frequency	Payment frequency is the number of payments within one year(integer) (1 - annually, 2 - semi-annually, 4 - quarterly, 12 - monthly)
I	Interest	Interest is the annual interest rate (real) used to discount the payments
INF	Increase frequency	Increase frequency is the increase frequency during one year (integer) (1 - annually, 2 - semi-annually, 4 - quarterly, 12 - monthly)
IT	Increase type	Increase type is the type of payment increasing (1 - geometric, 2 - linear/arithmetic)
IR	Increase rate	Increase rate is the annual rate (real) used for payment increase
PT	Payment type	1 - for payments at the beginning of the period, 0 - for payments at the end of the period
$\boldsymbol{\omega}$		Max age in LT
$\boldsymbol{v}$		$v = \frac{1}{1+I}$

7 TECHNICAL DOCUMENTATION

7.1 FORMULAS FOR 1 PERSON – LIFE TABLE

AF_lx (LT; X)

lx is in LT

AF_qx (LT; X)

$$q_x = \frac{l_x - l_{x+1}}{l_x} \tag{1.1}$$

AF_npx (LT; X; N)

n years survival probability

$$_np_x = \frac{l_{x+n}}{l_x} \tag{1.2}$$

AF_Dx (LT; X; I)

Commutation figure D_x

$$D_x = l_x v^x \tag{1.3}$$

AF_Cx (LT; X; I)

Commutation figure C_x

$$C_x = d_x v^{x+1} = (l_x - l_{x+1}) v^{x+1} \tag{1.4}$$

AF_Nx (LT; X; I)

Commutation figure N_x

$$N_x = \sum_{j=0}^{\omega-x} D_{x+j} \tag{1.5}$$

AF_Mx (LT; X; I)

Commutation figure M_x

$$M_x = \sum_{j=0}^{\omega-x} C_{x+j} \tag{1.6}$$

AF_Sx (LT; X; I)

Commutation figure S_x

$$S_x = \sum_{j=0}^{\omega-x} N_{x+j} \tag{1.7}$$

AF_Rx (LT; X; I)

Commutation figure R_x

$$R_x = \sum_{j=0}^{\omega-x} M_{x+j} \tag{1.8}$$

AF_Axn (LT; X; N; I)

Single netto premium for death and survival cover

$$A_{xn} = \ {}_nE_x + A_{xn}^{1} \tag{1.9}$$

AF_Exn (LT; X; N; I)

Single netto premium for pure endowment cover

$${}_nE_x = \frac{D_{x+n}}{D_x} \tag{1.10}$$

AF_A1xn (LT; X; N; I)

Single netto premium for pure death cover (term)

$$A^1_{xn} = \frac{M_x - M_{x+n}}{D_x} \tag{1.11}$$

AF_ann_xn (LT; X; N; PF; I; PT)

Present value of life annuity for limited number of years

- For payments at the beginning of the period:

$$\ddot{a}_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{1.12}$$

where

$$N = \left\{ b\frac{12}{PF}; b \in \langle 0, PF-1 \rangle;\ b \in \mathbb{Z} \right\}. \tag{1.13}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{1.14}$$

- For payments at the end of the period:

$$a_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{1.15}$$

where

$$N = \left\{ b\frac{12}{PF}; b \in \langle 1, PF \rangle;\ b \in \mathbb{Z} \right\}, \tag{1.16}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{1.17}$$

Note: Linear interpolation is made between $l_x$and l_{x+1}.

AF_kAxn (LT; X; N; K; I)

Single netto premium for death and survival cover with deferred period years of deferred period

$${}_{k|}A_{xn} = {}_{k|n}E_x + {}_{k|}A^1_{xn} \tag{1.18}$$

AF_kExn (LT; X; N; K; I)

Single netto premium for pure endowment cover with deferred period year of deferred period

$$ {}_{k|n}E_x = \frac{D_{x+k+n}}{D_x} \tag{1.19} $$

AF_kA1xn (LT; X; N; K; I)

Single netto premium for pure death cover (term) with deferred period year of deferred period

$$ {}_{k|}A^1_{xn} = \frac{M_{x+k} - M_{x+k+n}}{D_x} \tag{1.20} $$

AF_kann_xn (LT; X; N; K; PF; I; PT)

Present value of life annuity for limited number of years with deferred period years of deferred period

- For payments at the beginning of the period:

$$ {}_{k|}\ddot{a}_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{1.21} $$

where

$$ N = \left\{ b\frac{12}{PF}; b \in \langle 0, PF-1 \rangle;\ b \in \mathbb{Z} \right\}. \tag{1.22} $$

$$ pay_t = \begin{cases} \frac{1}{PF} \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 \qquad\qquad else. \end{cases} \tag{1.23} $$

- For payments at the end of the period:

$$ {}_{k|}a_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{1.24} $$

where

$$ N = \left\{ b\frac{12}{PF}; b \in \langle 1, PF \rangle;\ b \in \mathbb{Z} \right\}, \tag{1.25} $$

$$pay_t = \begin{cases} \frac{1}{PF}\frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 \qquad else. \end{cases} \tag{1.26}$$

Note: Linear interpolation is made between l_x and l_{x+1}.

AF_I_Axn (LT; X; N; I; INF; IT; IR)

Single netto premium for death and survival cover with increasing sum assured

$$(IA)_{xn} = \quad {}_n(IE)_x + (IA)^1_{xn} \tag{1.27}$$

AF_I_Exn (LT; X; N; I; INF; IT; IR)

Single netto premium for pure endowment cover with increasing sum assured

Increasetype is the type of increasing of payments:

- linear/arithmetic

$${}_n(IE)_x = \left[1 + (n \cdot INF - 1) \cdot \frac{IR}{INF}\right] \cdot \frac{D_{x+n}}{D_x} \tag{1.28}$$

- geometric

$${}_n(IE)_x = \left[1 + \frac{IR}{INF}\right]^{n \cdot INF - 1} \cdot \frac{D_{x+n}}{D_x} \tag{1.29}$$

AF_I_A1xn (LT; X; N; I; INF; IT; IR)

Single netto premium for pure death cover (term) with increasing sum assured

$$(IA)^1_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in M} inc_t \cdot d_{x+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{1.30}$$

where

$$M = \left\{ b \frac{12}{INF}; b \in \langle 1, INF \rangle;\ b \in \mathbb{Z} \right\}. \tag{1.31}$$

Increasetype is the type of increasing of payments:

- linear/arithmetic

$$inc_t = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, \quad (1.32)$$

- geometric

$$inc_t = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1. \quad (1.33)$$

Note: Linear interpolation is made between $d_x$and d_{x+1}.

AF_I_ann_xn (LT; X; N; PF; I; INF; IT; IR; PT)

Present value of life annuity for limited number of years with increasing payment

- For payments at the beginning of the period:

$$(I\ddot{a})_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \quad (1.34)$$

where

$$N = \left\{ b \frac{12}{freq}; b \in \langle 0, freq - 1 \rangle;\ b \in \mathbb{Z} \right\}, freq = max(INF, PF), \quad (1.35)$$

$$M = \left\{ b \frac{12}{INF}; b \in \langle 0, INF - 1 \rangle;\ b \in \mathbb{Z} \right\}. \quad (1.36)$$

$$pay_t = \begin{cases} & \frac{1}{PF} \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \quad (1.37)$$

- For payments at the end of the period:

$$(Ia)_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \quad (1.38)$$

where

$$N = \left\{ b \frac{12}{freq}; b \in \langle 1, freq \rangle;\ b \in \mathbb{Z} \right\}, freq = max(INF, PF), \quad (1.39)$$

$$M = \left\{ b \frac{12}{INF}; b \in \langle 1, INF \rangle;\ b \in \mathbb{Z} \right\} \quad (1.40)$$

$$pay_t = \begin{cases} & \frac{1}{PF} \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \quad (1.41)$$

Increasetype is the type of increasing of payments:

- linear/arithmetic

$$inc_{t\epsilon M} = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, inc_{t\notin M} = inc_{t-1}, \tag{1.42}$$

- geometric

$$inc_{t\epsilon M} = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1, inc_{t\notin M} = inc_{t-1}. \tag{1.43}$$

Note: Linear interpolation is made between $l_x$and l_{x+1}.

AF_I_kAxn (LT; X; N; K; I; INF; IT; IR)

Single netto premium for death and survival cover with increasing sum assured with deferred period years of deferred period

$$_{k|}(IA)_{xn} = \ _{k|n}(IE)_x + \ _{k|}(IA)^1_{xn} \tag{1.44}$$

AF_I_kExn (LT; X; N; K; I; INF; IT; IR)

Single netto premium for pure endowment cover with increasing sum assured with deferred period years of deferred period

Increasetype is the type of increasing of payments:

- linear/arithmetic

$$_{k|n}(IE)_x = \left[1 + (n \cdot INF - 1) \cdot \frac{IR}{INF}\right] \cdot \frac{D_{x+k+n}}{D_x} \tag{1.45}$$

- geometric

$$_{k|n}(IE)_x = \left[1 + \frac{IR}{INF}\right]^{n \cdot INF - 1} \cdot \frac{D_{x+k+n}}{D_x} \tag{1.46}$$

AF_I_kA1xn (LT; X; N; K; I; INF; IT; IR)

Single netto premium for pure death cover (term) with increasing sum assured with deferred period years of deferred period

$$(IA)^1_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in M} inc_t \cdot d_{x+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}}\right), \tag{1.47}$$

where

$$M = \left\{b\frac{12}{INF}; b \in \langle 1, INF\rangle;\ b \in \mathbb{Z}\right\}, \tag{1.48}$$

Increase type is the type of increasing of payments:

- linear/arithmetic

$$inc_t = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, \tag{1.49}$$

- geometric

$$inc_t = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1. \tag{1.50}$$

Note: Linear interpolation is made between $d_x$and d_{x+1}.

AF_I_kann_xn (LT; X; N; K; PF; I; INF; IT; IR; PT)

Present value of life annuity for limited number of years with increasing payment

- For payments at the beginning of the period:

$${}_{k|}(I\ddot{a})_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}}\right), \tag{1.51}$$

where

$$N = \left\{b\frac{12}{freq}; b \in \langle 0, freq-1\rangle;\ b \in \mathbb{Z}\right\}, freq = max(INF, PF), \tag{1.52}$$

$$M = \left\{b\frac{12}{INF}; b \in \langle 0, INF-1\rangle;\ b \in \mathbb{Z}\right\}. \tag{1.53}$$

$$pay_t = \begin{cases} & \frac{1}{PF}\frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{1.54}$$

- For payments at the end of the period:

$${}_{k|}(Ia)_{xn} = \frac{1}{l_x} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}}\right), \tag{1.55}$$

where

$$N = \left\{b\frac{12}{freq}; b \in \langle 1, freq\rangle;\ b \in \mathbb{Z}\right\}, freq = max(INF, PF), \tag{1.56}$$

$$M = \left\{b\frac{12}{INF}; b \in \langle 1, INF\rangle;\ b \in \mathbb{Z}\right\}, \tag{1.57}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{1.58}$$

Increase type is the type of increasing of payments:

- linear/arithmetic

$$inc_{t\epsilon M} = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, inc_{t\notin M} = inc_{t-1}, \tag{1.59}$$

- geometric

$$inc_{t\epsilon M} = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1, inc_{t\notin M} = inc_{t-1}. \tag{1.60}$$

Note: Linear interpolation is made between l_x and l_{x+1}.

7.2 Formulas for 2 persons - Life Table

AF_lxy (LT_X; LT_Y; X; Y)

l_x and l_y are in LT

$$l_{xy} = l_x l_y \tag{2.1}$$

AF_qxy (LT_X; LT_Y; X; Y)

$$q_{xy} = 1 - p_{xy} = 1 - p_x p_y = 1 - \frac{l_{x+1}}{l_x} \cdot \frac{l_{y+1}}{l_y} \tag{2.2}$$

AF_npxy (LT_X; LT_Y; X; Y; N)

n years survival probability for 2 lives

$${}_n p_{xy} = \frac{l_{x+n}}{l_x} \cdot \frac{l_{y+n}}{l_y} \tag{2.3}$$

AF_Dxy (LT_X; LT_Y; X; Y; I)

Commutation figure D_{xy}

$$D_{xy} = l_x l_y v^{\frac{x+y}{2}} \quad (2.4)$$

AF_Cxy (LT_X; LT_Y; X; Y; I)

Commutation figure C_{xy}

$$C_{xy} = d_{xy} v^{\frac{x+y}{2}+1} = (l_x l_y - l_{x+1} l_{y+1}) v^{\frac{x+y}{2}+1} \quad (2.5)$$

AF_Nxy (LT_X; LT_Y; X; Y; I)

Commutation figure N_{xy}

$$N_{xy} = \sum_{j=0}^{\omega-\max(x,y)} D_{x+j,y+j} \quad (2.6)$$

AF_Mxy (LT_X; LT_Y; X; Y; I)

Commutation figure M_{xy}

$$M_{xy} = \sum_{j=0}^{\omega-\max(x,y)} C_{x+j,y+j} \quad (2.7)$$

AF_Sxy (LT_X; LT_Y; X; Y; I)

Commutation figure S_{xy}

$$S_{xy} = \sum_{j=0}^{\omega-\max(x,y)} N_{x+j,y+j} \quad (2.8)$$

AF_Rxy (LT_X; LT_Y; X; Y; I)

Commutation figure R_{xy}

$$R_{xy} = \sum_{j=0}^{\omega-\max(x,y)} M_{x+j,y+j} \quad (2.9)$$

AF_Axyn (LT_X; LT_Y; X; Y; N; I)

Single netto premium for death and survival cover for 2 lives

$$A_{xyn} = \ {}_{n}E_{xy} + A^{1}_{xyn} \tag{2.10}$$

AF_Exyn (LT_X; LT_Y; X; Y; N; I)

Single netto premium for pure endowment cover for 2 lives

$${}_{n}E_{xy} = \frac{D_{x+n,y+n}}{D_{xy}} \tag{2.11}$$

AF_A1xyn (LT_X; LT_Y; X; Y; N; I)

Single netto premium for pure death cover (term) for 2 lives

$$A^{1}_{xyn} = \frac{M_{x,y} - M_{x+n,y+n}}{D_{xy}} \tag{2.12}$$

AF_ann_xyn (LT_X; LT_Y; X; Y; N; PF; I; PT)

Present value of life annuity for limited number of years for 2 lives

- For payments at the beginning of the period:

$$\ddot{a}_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+j-1+\frac{t}{12}, y+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{2.13}$$

where

$$N = \left\{ b\frac{12}{PF}; b \in \langle 0, PF-1 \rangle;\ b \in \mathbb{Z} \right\}. \tag{2.14}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.15}$$

- For payments at the end of the period:

$$a_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+j-1+\frac{t}{12}, y+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{2.16}$$

where

$$N = \left\{b\frac{12}{PF}; b \in \langle 1, PF\rangle;\ b \in \mathbb{Z}\right\}, \tag{2.17}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.18}$$

Note: Linear interpolation is made between l_x and l_{x+1}.

AF_kAxyn (LT_X; LT_Y; X; Y; N; K; I)

Single netto premium for death and survival cover for 2 lives with deferred period years of deferred period

$$_{k|}A_{xyn} = \ _{k|n}E_{xy} + \ _{k|}A^1_{xyn} \tag{2.19}$$

AF_kExyn (LT_X; LT_Y; X; Y; N; K; I)

Single netto premium for pure endowment cover for 2 lives with deferred period years of deferred period

$$_{k|n}E_{xy} = \frac{D_{x+k+n,y+k+n}}{D_{xy}} \tag{2.20}$$

AF_kA1xyn (LT_X; LT_Y; X; Y; N; K; I)

Single netto premium for pure death cover (term) for 2 lives with deferred period years of deferred period

$$_{k|}A^1_{xyn} = \frac{M_{x+k,y+k} - M_{x+k+n,y+k+n}}{D_{xy}} \tag{2.21}$$

AF_kann_xyn (LT_X; LT_Y; X; Y; N; K; PF; I; PT)

Present value of life annuity for 2 lives for limited number of years with deferred period years of deferred period

- For payments at the beginning of the period:

$$ {}_{k|}\ddot{a}_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+k+j-1+\frac{t}{12},y+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{2.22} $$

where

$$ N = \left\{ b\frac{12}{PF}; b \in \langle 0, PF-1 \rangle;\ b \in \mathbb{Z} \right\}. \tag{2.23} $$

$$ pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.24} $$

- For payments at the end of the period:

$$ {}_{k|}a_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot l_{x+k+j-1+\frac{t}{12},y+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{2.25} $$

where

$$ N = \left\{ b\frac{12}{PF}; b \in \langle 1, PF \rangle;\ b \in \mathbb{Z} \right\}, \tag{2.26} $$

$$ pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.27} $$

Note: Linear interpolation is made between l_x and l_{x+1}.

AF_I_Axyn (LT_X; LT_Y; X; Y; N; I; INF; IT; IR)

Single netto premium for death and survival cover for 2 lives with increasing sum assured

$$ (IA)_{xyn} = {}_{n}(IE)_{xy} + (IA)^{1}_{xyn} \tag{2.28} $$

AF_I_Exyn (LT_X; LT_Y; X; Y; N; I; INF; IT; IR)

Single netto premium for pure endowment cover for 2 lives with increasing sum assured

Increase type is the type of increasing of payments:

- linear/arithmetic

$$ {}_{n}(IE)_{xy} = \left[1 + (n \cdot INF - 1) \cdot \frac{IR}{INF} \right] \cdot \frac{D_{x+n,y+n}}{D_{xy}} \tag{2.29} $$

- geometric

$$ {}_n(IE)_{xy} = \left[1 + \frac{IR}{INF}\right]^{n \cdot INF - 1} \cdot \frac{D_{x+n,y+n}}{D_{xy}} \tag{2.30} $$

AF_I_A1xyn (LT_X; LT_Y; X; Y; N; I; INF; IT; IR)

Single netto premium for pure death cover (term) for 2 lives with increasing sum assured

$$ (IA)^1_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in M} inc_t \cdot d_{x+j-1+\frac{t}{12},y+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{2.31} $$

where

$$ M = \left\{ b\frac{12}{INF}; b \in \langle 1, INF \rangle;\ b \in \mathbb{Z} \right\}. \tag{2.32} $$

Increase type is the type of increasing of payments:

- linear/arithmetic

$$ inc_t = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, \tag{2.33} $$

- geometric

$$ inc_t = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1. \tag{2.34} $$

Note: Linear interpolation is made between d_x and d_{x+1}.

AF_I_ann_xyn (LT_X; LT_Y; X; Y; N; PF; I; INF; IT; IR; PT)

Present value of life annuity for 2 lives for limited number of years with increasing payment

- For payments at the beginning of the period:

$$ (I\ddot{a})_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+j-1+\frac{t}{12},y+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{2.35} $$

where

$$ N = \left\{ b\frac{12}{freq}; b \in \langle 0, freq-1 \rangle;\ b \in \mathbb{Z} \right\}, freq = max(INF, PF), \tag{2.36} $$

$$ M = \left\{ b\frac{12}{INF}; b \in \langle 0, INF-1 \rangle;\ b \in \mathbb{Z} \right\}. \tag{2.37} $$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.38}$$

- For payments at the end of the period:

$$(Ia)_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+j-1+\frac{t}{12}, y+j-1+\frac{t}{12}} \cdot v^{j-1+\frac{t}{12}} \right), \tag{2.39}$$

where

$$N = \left\{ b \frac{12}{freq} ; b \in \langle 1, freq \rangle ;\ b \in \mathbb{Z} \right\}, freq = max(INF, PF), \tag{2.40}$$

$$M = \left\{ b \frac{12}{INF} ; b \in \langle 1, INF \rangle ;\ b \in \mathbb{Z} \right\}, \tag{2.41}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.42}$$

Increase type is the type of increasing of payments:

- linear/arithmetic

$$inc_{t \epsilon M} = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, inc_{t \notin M} = inc_{t-1}, \tag{2.43}$$

- geometric

$$inc_{t \epsilon M} = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1, inc_{t \notin M} = inc_{t-1}. \tag{2.44}$$

Note: Linear interpolation is made between l_x and l_{x+1}.

AF_I_kAxyn (LT_X; LT_Y; X; Y; N; K; I; INF; IT; IR)

Single netto premium for death and survival cover for 2 lives with increasing sum assured with deferred period years of deferred period

$${}_{k|}(IA)_{xyn} = {}_{k|n}(IE)_{xy} + {}_{k|}(IA)_{xyn}^{1} \tag{2.45}$$

AF_I_kExyn (LT_X; LT_Y; X; Y; N; K; I; INF; IT; IR)

Single netto premium for pure endowment cover for 2 lives with increasing sum assured with deferred period years of deferred period

Increase type is the type of increasing of payments:

- linear/arithmetic

$$ {}_{k|n}(IE)_{xy} = \left[1 + (n \cdot INF - 1) \cdot \frac{IR}{INF}\right] \cdot \frac{D_{x+k+n,y+k+n}}{D_{xy}} \tag{2.46} $$

- geometric

$$ {}_{k|n}(IE)_{xy} = \left[1 + \frac{IR}{INF}\right]^{n \cdot INF - 1} \cdot \frac{D_{x+k+n,y+k+n}}{D_{xy}} \tag{2.47} $$

AF_I_kA1xyn (LT_X; LT_Y; X; Y; N; K; I; INF; IT; IR)

Single netto premium for pure death cover (term) for 2 lives with increasing sum assured with deferred period years of deferred period

$$ {}_{k|}(IA)^{1}_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in M} inc_t \cdot d_{x+k+j-1+\frac{t}{12}, y+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{2.48} $$

where

$$ M = \left\{ b\frac{12}{INF}; b \in \langle 1, INF \rangle;\ b \in \mathbb{Z} \right\}. \tag{2.49} $$

Increase type is the type of increasing of payments:

- linear/arithmetic

$$ inc_t = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, \tag{2.50} $$

- geometric

$$ inc_t = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1. \tag{2.51} $$

Note: Linear interpolation is made between d_x and d_{x+1}.

AF_I_kann_xyn (LT_X; LT_Y; X; Y; N; K; PF; I; INF; IT; IR; PT)

Present value of life annuity for 2 lives for limited number of years with increasing payment with deferred period years of deferred period

- For payments at the beginning of the period:

$$ {}_{k|}(I\ddot{a})_{xyn} = \frac{1}{l_{xy}} \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+k+j-1+\frac{t}{12}, y+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{2.52} $$

where

$$N = \left\{ b\frac{12}{freq}; b \in \langle 0, freq - 1 \rangle;\ b \in \mathbb{Z} \right\}, freq = max(INF, PF), \tag{2.53}$$

$$M = \left\{ b\frac{12}{INF}; b \in \langle 0, INF - 1 \rangle;\ b \in \mathbb{Z} \right\}. \tag{2.54}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.55}$$

- For payments at the end of the period:

$$_{k|}(Ia)_{xyn} = \frac{1}{l_{xy}} \cdot \sum_{j=1}^{n} \left(\sum_{t \in N} pay_t \cdot inc_t \cdot l_{x+k+j-1+\frac{t}{12}, y+k+j-1+\frac{t}{12}} \cdot v^{k+j-1+\frac{t}{12}} \right), \tag{2.56}$$

where

$$N = \left\{ b\frac{12}{freq}; b \in \langle 1, freq \rangle;\ b \in \mathbb{Z} \right\}, freq = max(INF, PF), \tag{2.57}$$

$$M = \left\{ b\frac{12}{INF}; b \in \langle 1, INF \rangle;\ b \in \mathbb{Z} \right\}, \tag{2.58}$$

$$pay_t = \begin{cases} \frac{1}{PF} & \frac{t \cdot PF}{12} \epsilon \mathbb{Z}, \\ 0 & else. \end{cases} \tag{2.59}$$

Increase type is the type of increasing of payments:

- linear/arithmetic

$$inc_{t \epsilon M} = inc_{t-1} + \frac{IR}{INF}, inc_1 = 1, inc_{t \notin M} = inc_{t-1}, \tag{2.60}$$

- geometric

$$inc_{t \epsilon M} = inc_{t-1} \cdot \left(1 + \frac{IR}{INF}\right), inc_1 = 1, inc_{t \notin M} = inc_{t-1}. \tag{2.61}$$

Note: Linear interpolation is made between l_x and l_{x+1}.

7.3 FORMULAS FOR GENERATION LIFE TABLE

The Formulas are the same like above, just first parameter is changed - GT (Generation Life Table) is written instead of parameter LT (Life table).

AF_qx_GT (GT; Year; X)

AF_lx_GT (GT; Year; X)

…

AF_qxy_GT (GT_X; GT_Y; Year_X; X; Year_Y; Y)

AF_lxy_GT (GT_X; GT_Y; Year_X; X; Year_Y; Y)

…

8 LITERATURE

CIPRA, Tomáš. *Finanční a pojistné vzorce*. Praha: Grada Publishing, a. s., 2006. ISBN 80-247-1633-X.