



# Claims Reserve Calculator

## User Guide

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## 1 INTRODUCTION

The Claims Reserve Calculator is a software tool developed in MS Excel. Its goal is to help actuaries with the process of claims reserving that is crucial especially for non-life insurance companies. We believe that this application will make your actuarial work **easier, more comfortable, faster and more complex**.

It is based on the analysis of the aggregated data expressed in the form of a development triangle and the extrapolation of the aggregated values to the future using variety of development factors.

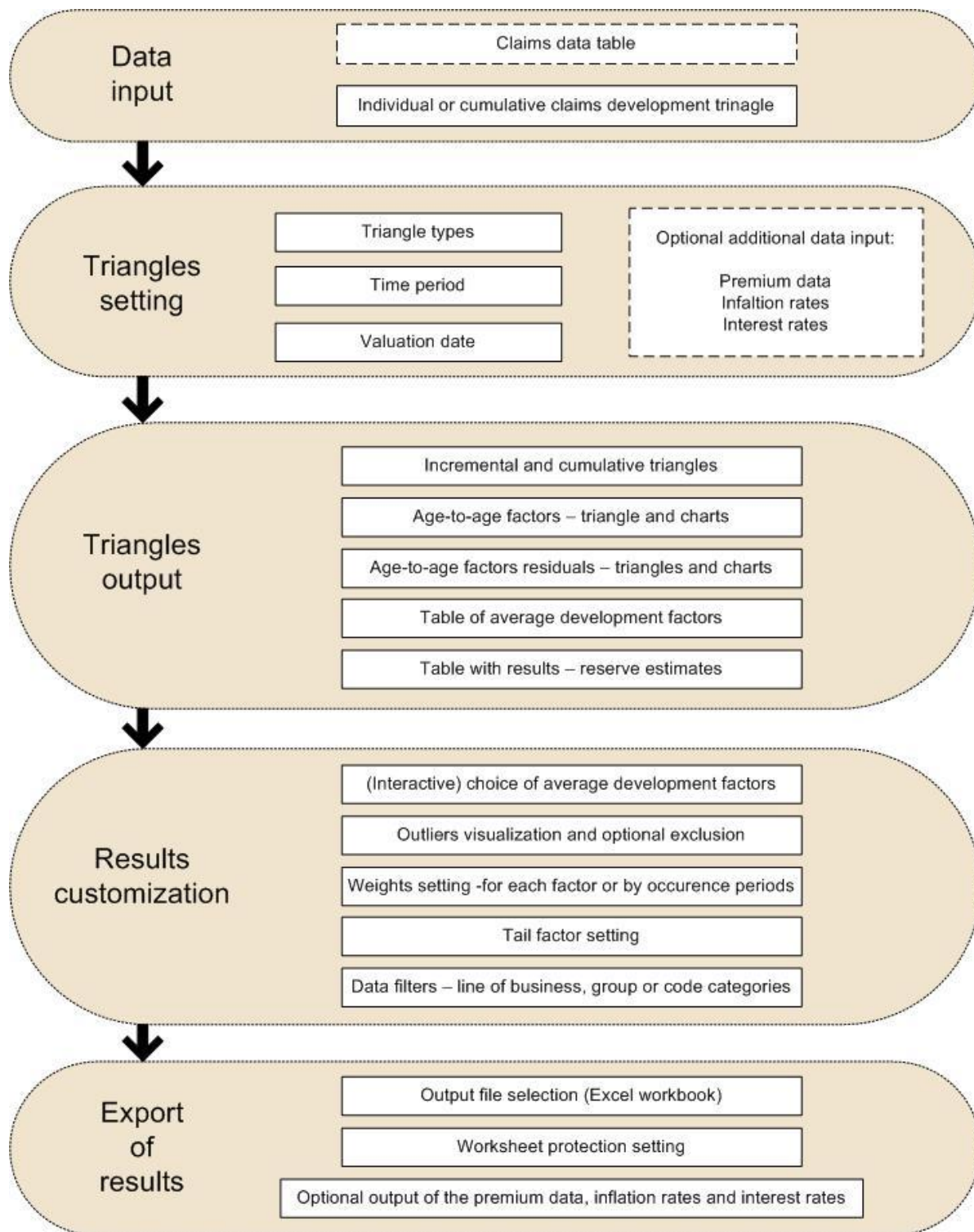
Its main functions are as follows:

- To **extract the appropriate data** from the claims data table **and arrange them in a form of development triangle**, according to the user's requirements. The customization consists mainly in the choice of:
  - the length of the triangle time period – 1 month, 1/4 year, 1/2 year, 1 year;
  - the triangle types to be calculated – 5 different methods of a data aggregation (Payment triangle – occurrence vs. booking, Incurred triangle – occurrence vs. reporting, Incurred triangle – occurrence vs. booking, Number of claims paid out triangle, Number of reported claims triangle).
- To calculate **several variants of average development factors** with their immediate application to the extrapolation of historical triangles.

Other useful functions of the application:

- **inflation adjustment** of the data;
- **discounting** of extrapolated values;
- **loss ratio** triangles calculation;
- **Bornhuetter-Fergusson** method application;
- **graphical representation** of the individual development factors and residuals;
- **identification of the extreme values** of individual development factors;
- **export** of results into the separate Excel workbook.

## PROCESS FLOW DIAGRAM



## NEW FEATURES IN VERSION 1.08

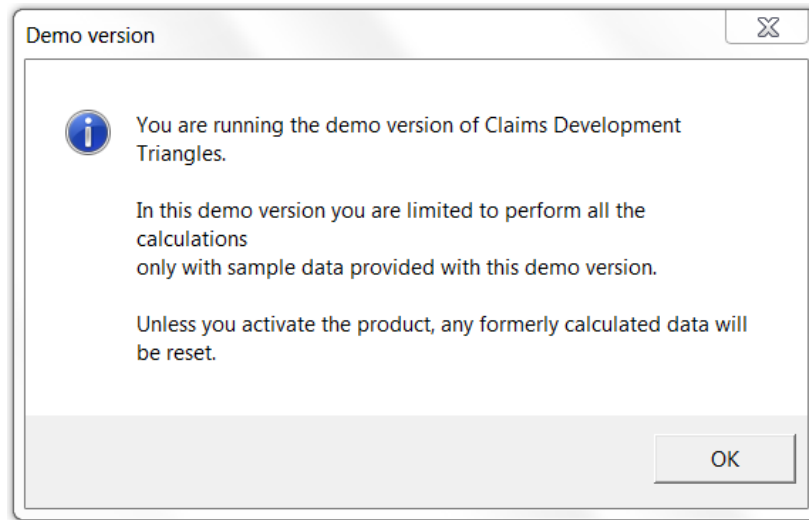
**Implementation of Bornhuetter-Fergusson (B-F) method:** Projection from payment and incurred triangles can be combined with prior estimation based on ultimate loss ratios.

**Extension of claims data:** New column added for number of paid claims. This enables to use the aggregated data on the input and calculate the triangle with number realized payments.

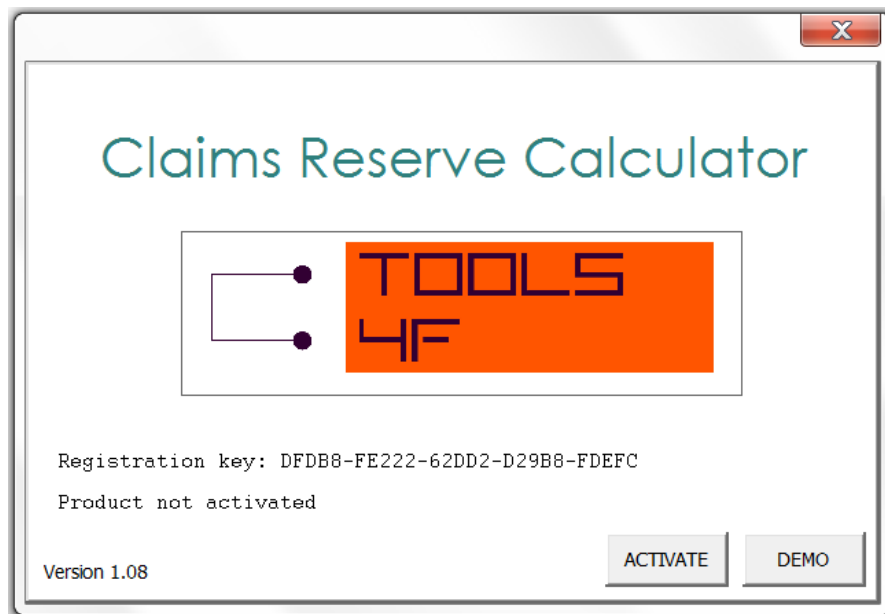
**Premium defined on LOB, Group and Code level:** Premium is defined on the same level as is used in the claims data table. This enables to correctly apply the filter on calculated results, including loss ratios and B-F method results.

Furthermore, several adjustments have been made to make the tool more user-friendly.

## 2 DEMO VERSION AND ACTIVATION



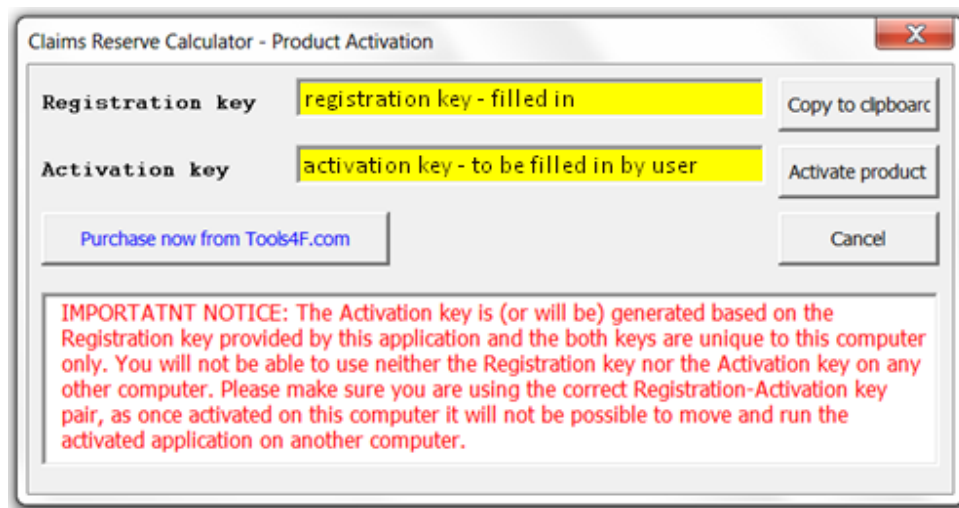
After opening the application, you will be informed about running the demo version. Now you can only run the demo version until you insert the valid product key. In the demo version you can only use the provided demo data which are included in the file you downloaded from the website. Click “OK” to continue.



Now only the demo version is active. If you want to run the full version of the application, click on the button “Activate” otherwise continue in the demo version by clicking on the button “Demo”.

To get the Activation key, go to the web site [www.tools4f.com](http://www.tools4f.com) and purchase the application. Once the license is ordered and paid, you will be sent the Activation key to your e-mail. After receiving the Activation key, copy it to the box “Activation key” and you can activate the full version of this application with the button “Activate product”.

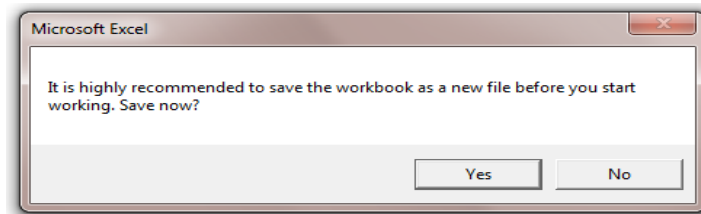
NOTE: The Activation key is generated based on the Registration key provided by this application and the both keys are unique to this computer only. You will not be able use neither the Registration key nor the Activation key on any other computer. Please make sure you are using the correct Registration - Activation key pair, as once activated on your computer it will not be possible to move and run the activated application on another computer.



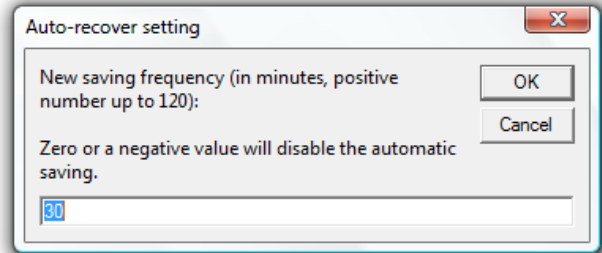
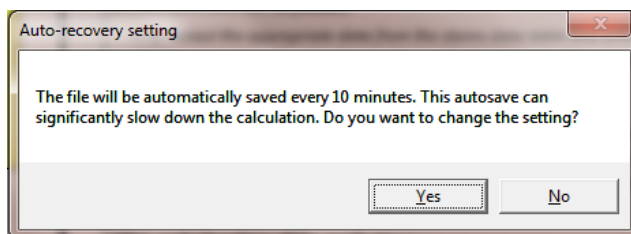
From now on, you can use all the functions of the application.

## 3 USING THE APPLICATION

After opening the file you will be asked if you would like to save the workbook as a new file. It is highly recommended to do so, since the original file would stay unchanged and ready to be used without the need of any reset procedure.



Next you will be asked if you would like to change the auto-recovery setting. Usually the MS Office application is set to save the working file automatically with a specified frequency. This 'autosaving' procedure can slow down the calculation, especially if you work with a large claims input data. Thus you are given the possibility to change the setting to make the whole process faster. You can either change the auto-save frequency or even disable the automatic saving.



### 3.1 CLAIMS DATA SPECIFICATION

At the beginning only the *InitialSetting* sheet is visible. Here you can see the short description of the application and the *Claims data specification* form. The *Complete reset of the application* button under the form can be used to reset the application when you want to start to work with new data.

Claims data specification

Choose the form of the claims data input and use the **CREATE INPUTS** button to confirm your choice. The claims data can be represented either as a data table (with a pre-specified columns structure) or directly in the form of a claim triangle.

☒ Get data from a text file
 ☐ Fill in the data table
 ☐ Fill in the triangle

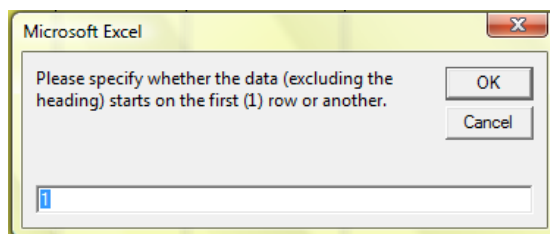
CREATE INPUTS



Select the preferred option of the data input form and confirm your choice using the *Create Inputs* button. Generally, there are two default types of the claims data representation – the data table or directly the development triangle.

## 3.2 DATA TABLE

To fill in the data table you can import the data from a text file (choose the first option in the *Claims data specification* form) or fill in the data table directly in Excel (choose the second option in the form). If you choose to import the data from a file, first the file browser is opened to let you choose the file. (In this case, the file has to have a structure with the semicolon as a separator. If you want to import the data from a text file with different structure, choose the second option in the *Claims data specification* form and import the data by yourself using the Excel import function.) A warning message can appear in case that the selected text file comprises more than 200,000 lines (records). Due to the MS Office possibilities large files can take a long time to be processed. If you need to work with large datasets and you do not want to wait several minutes for the results we recommend you to pre-aggregate the data using some other software tool. Second, you are asked to specify the number of the row where the actual data starts. This ensures that only the claims data are imported, without any heading or other information.

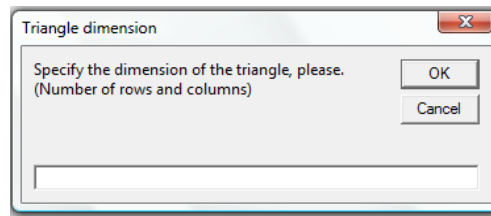


For both options you have to respect the pre-specified structure of columns. The complete data table would be comprised of these columns: Line of business, Claim ID, Occurrence date, Reporting date, Booking date, Claims payment, Number of paid claims, Outstanding reserve change, Code and Group. For each row it is necessary to fill in at least the *Occurrence date* column and *Reporting date* or *Booking date* column. The requirement of filling in the other columns depends on the choice of the triangle types to be calculated (see the chapter 3.4 Triangles specification). The usual operations with the data are allowed - you can apply the data filter<sup>1</sup>, change the formatting, delete or insert rows, etc. Before continuing you should delete all the labels of the columns you have not used. Use the *Reset* button if you want to clear the whole table. Once satisfied with the data table, click on the *Go to triangles setting* button to continue with the triangles specification (chapter 3.4).

<sup>1</sup> It is also possible to filter the data according to the Line of business, Group and Code categories later, directly on the sheets with calculated triangles. See chapter 3.6.

### 3.3 TRIANGLE

The third option in the *Claims data specification* form, to fill in the triangle, is to be used if you have the data already aggregated in the triangle form. Selecting this option you will be asked to specify the dimension of the triangle (in this version only the square matrix is possible to be processed by the application, i.e. the specified dimension determines the number of rows as well as the number of columns).



After the click on the *OK* button a new sheet (*UserTriangle*) is opened.

Export

**Time specification**

Valuation date: 12-2015 Calendar

Period: 1 year

**Triangle types specification**

Inflation adjustment: NO Fill in the data table

Loss ratio: NO Fill in the data table

Bornhuetter Ferg: NO Fill in the data table

Discounting: NO Fill in the data table

**Results specification**

Tail: 1

Tail discount factor: 1

Inflation: Ignored

**Input triangle specification**

Select... Monetary

Refresh

**CREATE THE TRIANGLES**

Reset

Incremental triangle - inflation ignored		Development period					
Occurrence period		0	1	2	3	4	5
PeriodStart	PeriodEnd						
2010-01	2010-12						
2011-01	2011-12						
2012-01	2012-12						
2013-01	2013-12						
2014-01	2014-12						
2015-01	2015-12						

Cumulative triangle - inflation ignored		Development period					
Occurrence period		0	1	2	3	4	5
PeriodStart	PeriodEnd						
2010-01	2010-12						
2011-01	2011-12						
2012-01	2012-12						
2013-01	2013-12						
2014-01	2014-12						
2015-01	2015-12						

You can fill in either the incremental or the cumulative triangle. Your choice has to be specified in the *Input triangle specification* form. The second item of this form (*Monetary* or *Count*) determines whether the data in the triangle will be processed as represented in the monetary or non-monetary units. The forms at the top of the sheet specify the additional parameters for the triangle. Their structure is similar as for the triangles created from the claims data table and are described in the following sections (3.4 Triangles specification, 3.5 Optional data and 3.9 Results). After the complete specification of all required data you can start the calculation of required triangles (and summary results) using the *CREATE THE TRIANGLES* button. To reset all

the input forms and clear all the data in the sheet (both the input and the calculated data) use the *Reset* button.

### 3.4 TRIANGLES SPECIFICATION

On the *TrianglesSetting* sheet you are asked to specify the time parameters (the valuation date and the time period for triangle rows and columns) and the types of triangles to be calculated. In case you choose to calculate any inflation adjusted, loss ratio or discounted triangle you have to fill in the respective input data (see Chapter 3.5 Optional data).

#### Time parameters

Time specification				
Valuation date:	12-2011	Calendar	Demo data - optimal setting	CONFIRM NEW SETTING
Period:	1 month			

To specify the valuation date you can either choose the date from the calendar or directly write the date into the cell. The last day of the chosen month is taken as the valuation date no matter which specific day you choose. The difference between the valuation date and the oldest record within the input data (in the *Occurrence date* column) has to be greater than the specified time period's length. For example, if the occurrence date of the oldest record within the claims data is February 22, 2011 and the chosen time period is 1/4 year, the valuation month cannot be older than May 2011 (again, only the respective months are taken into account, no matter the specific day within the month). This ensures that the development triangles would be at least of dimension 2 so that a future development estimation is possible. The time periods to choose from are: 1 month, 1/4 year, 1/2 year, 1 year. You have to press the *Confirm new setting* button to make your setting valid.

#### Triangle types

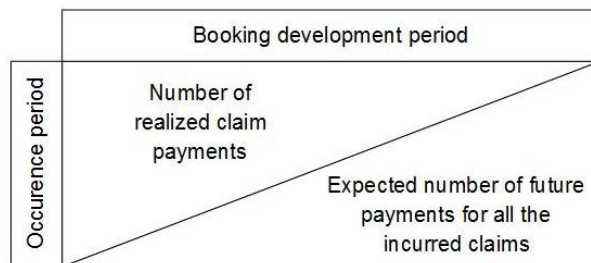
Triangle types				
Number of claims paid out	Number of reported claims	Payment triangle (occurrence vs. booking)	Incurred triangle (occurrence vs. reporting)	Incurred triangle (occurrence vs. booking)
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/> (none / all)	<input type="checkbox"/> (none/all)	<input type="checkbox"/> (none/all)
		<input type="checkbox"/> Inflation ignored	<input type="checkbox"/> Inflation ignored	<input type="checkbox"/> Inflation ignored
		<input type="checkbox"/> Inflation adjusted	<input type="checkbox"/> Inflation adjusted	<input type="checkbox"/> Inflation adjusted
		<input type="checkbox"/> Loss ratio	<input type="checkbox"/> Loss ratio	<input type="checkbox"/> Loss ratio
		<input type="checkbox"/> Discounting	<input type="checkbox"/> Discounting	<input type="checkbox"/> Discounting
		<input type="checkbox"/> Bornhuetter Ferguson	<input type="checkbox"/> Bornhuetter Ferguson	<input type="checkbox"/> Bornhuetter Ferguson

There are five basic types of triangles you can choose from. The first two of them are of the 'count' type. In the first triangle (*Number of claims paid out*) the claims data are aggregated

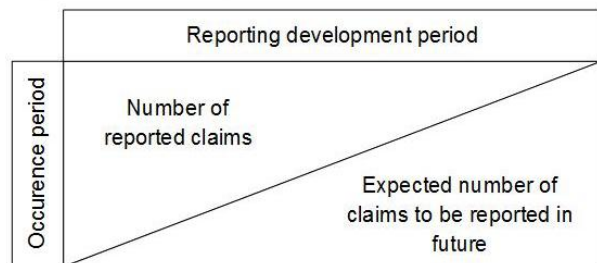
according to the *Occurrence date* and *Booking date* while in the second triangle (*Number of reported claims*) the aggregation is made according to the *Occurrence date* and *Reporting date*. To create the first triangle, data from *Number of paid claims* column in claims data table is summed up, while for the second triangle, the number of records with a unique *Claim ID* is counted within each of the aggregation groups.

The last three triangle types are of the ‘monetary’ type – representing currency units. For all of them you can work either with the basic (inflation ignored) or the inflation adjusted triangle (or with both of them). Apart of this choice you can decide if you want to calculate and display the loss ratio triangles and the triangles with extrapolated values discounted to the valuation date. If any of these options is chosen you have to fill in the respective data (see the next chapter). The triangles’ structure is presented by following schemes.

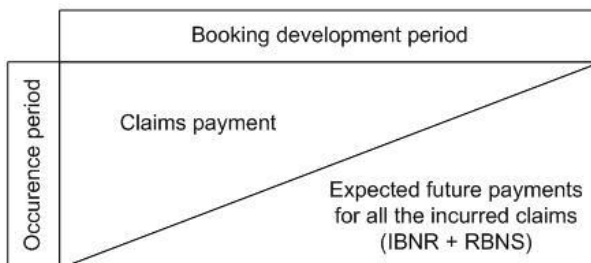
Claim payments triangle



Reported claims triangle



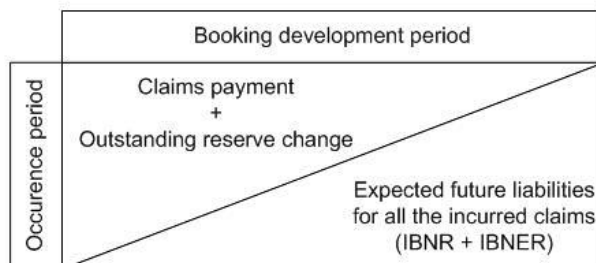
Payment triangle (occurrence vs. booking)



Incurred triangle (occurrence vs. reporting)



Incurred triangle (occurrence vs. booking)



### 3.5 OPTIONAL DATA

If you want to calculate the triangles of loss ratios, the inflation adjusted triangles or the triangles with discounted extrapolated data, or if you want to use the Bornhuetter-Fergusson method, you have to fill in the respective data into the data tables on the *OtherData* sheet. This sheet is opened after a click on any of the *Fill in the data table* buttons either on the *TrianglesSetting* or the *UserTriangle* sheet.

#### Premium table

Premium table											Calculate premium	
Fill in either the monthly premium data (in the left part of the table) or directly the premium values for each occurrence period. You can use the "Calculate premium" button to pre-fill the PeriodStart and PeriodEnd columns, according to the previously chosen valuation date and input claims data.												
Year	Month	Line.of.business	Group	Code	Year-Month	Premium	PeriodStart	PeriodEnd	Line.of.business	Group	Code	Premium

To calculate the loss ratio triangles or use the Bornhuetter-Fergusson method you have to fill in the *Premium table*. In the left part of the table you can fill in the monthly premium data (for each LOB, Group and Code category) and then use the *Calculate premium* button to calculate (by the aggregation of monthly data) the values for all the occurrence periods. Alternatively, in the right part of the table you can fill in directly the premium values for each combination of occurrence period and LOB, Group and Code categories – in this case, it is recommended to use the *Calculate premium* button to pre-fill in the *PeriodStart*, *PeriodEnd*, *Line.of.business*, *Group* and *Code* columns prior to your premium specification (if it has not been automatically filled yet). This button serves also as a reset for the right part of the table. It is assumed that the premium values are in the same monetary units as for the claims data.

#### Inflation rates table

Inflation rates table						Calculate inflation adjustment factors
Fill in either the yearly inflation rates in the left part of the table (as per annum rates) or directly the inflation adjustment factors for each occurrence period (see the User Guide for more detailed specification). You can use the "Calculate inflation adjustment factors" button to pre-fill the PeriodStart and PeriodEnd columns, according to the previously chosen valuation date and input claims data.						
Year	Inflation rate	PeriodStart	PeriodEnd	EdgeFactors	Inflation adjustment factors	

To work with the inflation adjusted triangles you have to fill in the *Inflation rates table*. In the left part of the table the first column represents the calendar year, the second the respective rate of inflation expressed as a per annum rate. The *EdgeFactors* represent the adjustment factors from the beginning/end of the respective past/future period to the valuation date. To the triangle data, the *Inflation adjustment factors*, calculated as a geometric average of the *PeriodEnd* and *PeriodStart* factors, are applied. More details and the formulae can be found in the Technical documentation (Chapter 4 of this Guide). To fill in the table, you can either completely fill in the left part of the table and then calculate the adjustment factors using the *Calculate inflation adjustment factors* button or fill in the *Inflation adjustment factors* directly. In the second case, use the button to fill in the *PeriodStart* and *PeriodEnd* columns. The button serves also as a reset for the right part of the table.

### Discount factors table

Discount factors table				Calculate discount factors	
Fill in either the yearly interest rates in the left part of the table (as per annum spot rates, valid at valuation date) or directly the discount factors for each occurrence period (see the User Guide for more detailed specification). You can use the "Calculate discount factors" button to pre-fill the PeriodStart and PeriodEnd columns, according to the previously chosen valuation date and input claims data.					
Years to maturity	Interest rate	PeriodStart	PeriodEnd	PeriodEnd discount factor	Discount factor
0	0%				

The discount factors are demanded if you want to calculate the triangles with discounted extrapolated values. In the left part of the *Discount factors table* you can fill in the interest rates (in percent per annum) as spot rates valid at the valuation date. By default, the interest rate with zero time to maturity is set to 0 %. Use the *Calculate discount factors* button to calculate the right part of the table (for the details of the calculation, with respective formulae, see the Technical documentation – Chapter 4). You can also fill in directly the discount factors for each time period – in this case, use the *Calculate discount factors* button to fill in the *PeriodStart* and *PeriodEnd* columns prior to your discount factors specification. The button serves also as a reset for the right part of the table.

## 3.6 OUTPUTS

The structure of the output partially depends on the selected option of the claims data input. For the data represented as a data table a new sheet is created for each selected triangle type. For the data input in the form of a triangle the calculations are proceeded directly on the *UserTriangle* sheet. In general, the structure of the output is as follows:

- Results specification form and the summary table of results – see chapter 3.9. Results.
- Pivot table filter – only for the input data in the form of claims data table. Compared to the data filter possibly applied directly on the claims data table, this filter works interactively, i.e. all the displayed triangles and the values in the result table are recalculated with the change of this filter. Adjust the filter and use the *Refresh* button to recalculate the results.
- Calculated triangles and average development factors – see chapters 3.7 Output triangles and 3.8 Average factors choice.

### 3.7 OUTPUT TRIANGLES

The maximum possible output includes these development triangles, tables and charts (the abbreviations in parenthesis are used in the formulae in chapter 4, Technical documentation):

- Inflation ignored triangles
  - Incremental triangle (Bas)
  - Cumulative triangle (Bas)
  - Discounted values – incremental triangle (Disc)
  - Discounted values – cumulative triangle (Disc)
  - Loss ratio – incremental triangle (LR)
  - Loss ratio – cumulative triangle (LR)
  - Age-to-age factors triangle
    - Development factors charts – one with the Development period and the second with the End of the origin period on the horizontal axis.
    - Development factors trendlines chart.
  - Average development factors table
  - Age-to-age factors residuals triangle
    - According to the origin period
      - Residuals charts – one with the Development period and the second with the End of the origin period on the horizontal axis.
    - According to the calendar period
      - Residuals chart
- Inflation adjusted triangles
  - Incremental triangle (IA)
  - Cumulative triangle (IA)
  - Incremental triangle – nominal values (Nom)
  - Cumulative triangle – nominal values (Nom)
  - Discounted values – incremental triangle (IADisc)
  - Discounted values – cumulative triangle (IADisc)
  - Loss ratio – incremental triangle (IALR)
  - Loss ratio – cumulative triangle (IALR)
  - Age-to-age factors triangle
    - Development factors charts – one with the Development period and the second with the End of the origin period on the horizontal axis.
  - Average development factors table
  - Age-to-age factors residuals triangle
    - According to the origin period
      - Residuals charts – one with the Development period and the second with the End of the origin period on the horizontal axis.
    - According to the calendar period
      - Residuals chart



### 3.8 AVERAGE FACTORS CHOICE

The *AVERAGE DEVELOPMENT FACTORS* table is divided into the three parts. In the first part you can choose the type of average development factors that you want to apply on the historical data to get the extrapolation – choose from the *Selected option* list. All the extrapolated values as well as the results are calculated directly after your choice.

AVERAGE DEVELOPMENT FACTORS	
Selected option	Smpl
Extreme values parameter	1,1

The *Extreme values parameter* serves as a parameter to identify the extreme values of (individual) age-to-age factors. The values that are greater/smaller than the selected average factor plus/minus the respective standard error multiplied by the *Extreme values parameter* are marked (with red colouring) as extreme values. There is no effect on the calculated results unless the *(De)Ignore Selected* button is used. To ignore the extreme values (or any other), first the individual factors for which you want to change the 'ignorance status' have to be chosen by the multiple selection of the respective cells. After clicking on the button, the selected cells are marked by changed interior pattern (criss-cross pattern) and their weights in the *General weights* triangle are set to zero. The selection can be cancelled by the *Clear* button.

Age-to-age factors - inflation ignored					
Occurrence period		Development period			
PeriodStart	PeriodEnd	0->1	1->2	2->3	3->4
2000-01	2000-12	2.92	1.04	1.01	1.00
2001-01	2001-12	1.80	1.05	1.00	
2002-01	2002-12	2.00	1.26		
2003-01	2003-12	1.67			

The second part of the *DEVELOPMENT FACTORS* table is the *General weights* triangle.

General weights					
Occurrence period		Development period			
PeriodStart	PeriodEnd	0->1	1->2	2->3	3->4
2001-01	2001-12	1	1	1	1
2002-01	2002-12	1	1	1	
2003-01	2003-12	1	1		
2004-01	2004-12	1			

Here you can specify the weights individually for each of the age-to-age factors. These weights are applied to all of the automatically calculated average types. By default, all the weights are set to one. The weights are automatically normalized so that any number can be used here. Using the *Ignore selected factors* button (see further), the respective weights are automatically set to zero.

The last part is the table of calculated average factors and respective standard errors. The first two average types are calculated from all of the age-to-age factors. The next five average types take into account only the most recent data – maximum last N (N = 1 to 5) factors from each column of the age-to-age factors triangle. For the next two average types you can specify the weight for each row of the age-to-age factors triangle (i.e. for each occurrence period). Using these weights, the simple and volume weighted average of all the individual factors are calculated. The last option how to specify the development factors to be applied for the future values extrapolation is to set the factors' values directly (the *User defined factors* option). For more detailed description of the individual average calculation see the Technical documentation (Chapter 4). The actually selected average type used for extrapolation is identified by the yellow colouring of the cells with the factors' values.

Average factors values and standard errors				
Average type	0->1	1->2	2->3	3->4
Simple average	1,65822	1,13095	1,05653	1,01898
Std. error	0,38278	0,04533	0,01479	
Volume weighted average	1,56506	1,13774	1,05516	1,01898
Std. error	0,30536	0,04782	0,01472	
Last 1 factor	2,27868	1,19257	1,04175	1,01898
Std. error				
Last 2 volume weighted average	1,83158	1,14558	1,05516	1,01898
Std. error	0,27655	0,05343	0,01472	
Last 3 volume weighted average	1,66265	1,13774	1,05516	1,01898
Std. error	0,2988	0,04782	0,01472	
Last 4 volume weighted average	1,56506	1,13774	1,05516	1,01898
Std. error	0,30536	0,04782	0,01472	
Last 5 volume weighted average	1,56506	1,13774	1,05516	1,01898
Std. error	0,30536	0,04782	0,01472	
Occurrence period weights - simple average	1,65822	1,13095	1,05653	1,01898
Std. error	0,38278	0,04533	0,01479	
Occurrence period weights - volume weighted average	1,56506	1,13774	1,05516	1,01898
Std. error	0,30536	0,04782	0,01472	
Occurrence period				
PeriodStart	PeriodEnd			
2001-01	2001-12	1		
2002-01	2002-12	1		
2003-01	2003-12	1		
2004-01	2004-12	1		
User defined factors				
Std. error				
Selected option	1,65822	1,13095	1,05653	1,01898
Std. error	0,38278	0,04533	0,01479	

### 3.9 RESULTS

The summarized results are displayed at the top of the sheet with triangles, below the *Results specification* form.

Results specification	
Tail:	1,05
Tail discount factor:	0,96
Inflation:	Ignored

In this form you can set the tail factor as well as the discounting factor for the tail value. You can also choose whether the results are calculated from the original (inflation ignored) or the inflation adjusted triangles. All of these options work interactively, i.e. the results are recalculated immediately after you make any change in the form.

The complete table with results can look like this:

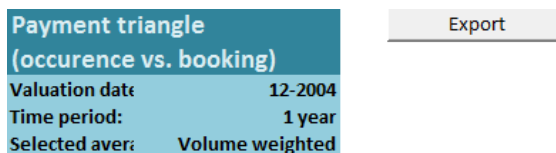
RESULTS																	
Occurrence period		No discounting						Discounted values				Bornhuetter Ferguson method					
PeriodStart	PeriodEnd	Latest incurred	Triangle ultimate	Triangle extrapolated	Tail value	extrapolated	Total ultimate	Triangle extrapolated	Tail value	extrapolated	Total	Premium	Prior ultimate loss ratio	Prior ultimate	Selected CDF	Expected future	Ultimate derived
2000-01	2000-12	1 703 079	1 703 079	0	0	0	1 703 079	0	0	0	2 503 840	70%	1 752 688	1,00	0	1 703 079	
2001-01	2001-12	2 160 465	2 164 258	3 793	0	3 793	2 164 258	3 774	0	3 774	3 293 040	70%	2 305 128	1,00	4 040	2 164 505	
2002-01	2002-12	2 738 260	2 807 717	69 457	0	69 457	2 807 717	69 017	0	69 017	2 384 028	70%	1 668 820	1,03	41 283	2 779 543	
2003-01	2003-12	1 253 017	1 381 341	128 324	0	128 324	1 381 341	126 997	0	126 997	2 850 342	70%	1 995 239	1,10	185 354	1 438 371	
2004-01	2004-12	1 109 218	2 828 988	1 719 770	0	1 719 770	2 828 988	1 705 337	0	1 705 337	3 584 035	70%	2 508 825	2,55	1 525 139	2 634 357	
Total		8 964 039	10 885 384	1 921 345	0	1 921 345	10 885 384	1 905 126	0	1 905 126	14 615 285		10 230 700		1 755 817	10 719 856	

The discounted values are displayed only for monetary data and only if the triangles with discounted values are calculated. The categories that are calculated are as follows: Latest incurred value, Triangle ultimate value, Triangle extrapolated value (nominal and discounted), Tail value (nominal and discounted), Total extrapolated value (nominal and discounted), Total ultimate value. All the formulae can be found in the Technical documentation (Chapter 4).

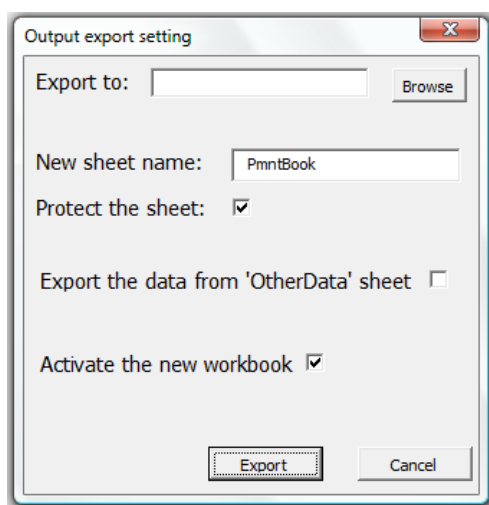
The right part of the table provides the results for Bornhuetter-Fergusson method (available only for monetary data). The first three columns (with yellow background) are to be filled with the input parameters for the method: Premium (by default, this is linked to the Premium input table), Prior ultimate loss ratio, Prior ultimate (by default calculated from premium volume and ultimate loss ratio, but can be also filled in directly by the user). In the next columns, results are calculated: Cumulative development factors (CDF) from the triangles, Expected future and Ultimate derived volume.

### 3.10EXPORT OF THE OUTPUT

The possibility of output export is provided through the *Export* button that you can find at the top of all the sheets with calculated triangles.



The *output export setting* form is opened after the click on this button.



Here you can choose the file to export the results to. Using the file browser you can either choose the existing file or create a new one. The other options in the form specify the export setting. If you choose to protect the sheet, the protection (without any password) will be applied. It is also possible to export the data tables from the *OtherData* sheet. In that case the table values will be copied at the end of the sheet (under all of the triangles). If you want to check the exported sheet, choose to activate the new workbook. Otherwise the export file will be saved and automatically closed.

## 4 TECHNICAL DOCUMENTATION

In this section of the Guide you can find mainly the formulae used by the application, in some cases with a little bit more detailed explanation.

### 4.1 NOTATION

First we have to introduce the notation used within the Technical documentation.

$n$  ... triangle dimension (a number of rows as well as a number of columns);

$TrglTypes = \{Bas, Disc, LR, IA, Nom, IADisc, IALR\}$  ... set of the triangle types;

$AvgTypes = \{Smpl, Vwtd, Last1, Last2, Last3, Last4, Last5, OccPerSmpl, OccPerVwtd, UserDefined, Selected\}$  ... set of the development factors average types;

$IlorIA = \{II, IA\}$  ... set of the inflation manipulation types (ignored, adjusted);

$ResTypes = \{OccPer, CalPer\}$  ... set of the residuals types (occurrence, calendar period);

$\{C_{i,j}^{Trgl} | i = 1, \dots, n; j = 1, \dots, n; Trgl \in TrglTypes\}$  ... incremental triangles;

$\{D_{i,j}^{Trgl} | i = 1, \dots, n; j = 1, \dots, n; Trgl \in TrglTypes\}$  ... cumulative triangles;

$\{\lambda_{i,j}^I | i = 1, \dots, n-1; j = 1, \dots, n-i; I \in IlorIA\}$  ... age-to-age factors' triangles;

$\{\hat{\lambda}_j^{I,Avg} | j = 1, \dots, n-1; I \in IlorIA, Avg \in AvgTypes\}$  ... average development factors;

$\{\hat{\sigma}_j^{I,Avg} | j = 1, \dots, n-1; I \in IlorIA, Avg \in AvgTypes\}$  ... dev. factors standard errors;

$\{r_{i,j}^{I,Res} | i = 1, \dots, n-1; j = 1, \dots, n-1; I \in IlorIA, Res \in ResTypes\}$  ... residuals;

$\{w_{i,j}^I | i = 1, \dots, n-1; j = 1, \dots, n-i; I \in IlorIA\}$  ... general weights triangles;

$\{wOcc_i^I | i = 1, \dots, n-1; I \in IlorIA\}$  ... occurrence periods' weights;

$\{P_i | i = 1, \dots, n\}$  ... premium values;

$\pi_Y$  ... per annum inflation rate for calendar year  $Y$ ;

$IAF_t$  ... inflation adjustment factor for  $t$  periods after valuation date ( $t = -n+1, \dots, n-1$ );

$IR_T$  ... per annum interest rate (for  $T$  years to maturity);

$DF_i$  ... discount factor for occurrence period  $i$  ( $i = n+1, n+2, \dots, 2n$ );

$\{LI_i | i = 1, \dots, n\}$  ... latest incurred values;

$\{TrE_i | i = 1, \dots, n\}$  ... triangle extrapolated values;

$\{TrU_i | i = 1, \dots, n\}$  ... triangle ultimate values;

$\{Tail_i | i = 1, \dots, n\}$  ... tail values;

$\{TE_i \mid i = 1, \dots, n\}$  ... total extrapolated values;  
 $\{TU_i \mid i = 1, \dots, n\}$  ... total ultimate values;  
 $\{TrE_i^{Disc} \mid i = 1, \dots, n\}$  ... discounted triangle extrapolated values;  
 $\{Tail_i^{Disc} \mid i = 1, \dots, n\}$  ... discounted tailvalues;  
 $\{TE_i^{Disc} \mid i = 1, \dots, n\}$  ... discounted total extrapolated values;  
 $\lambda_{Tail}$  ... tail factor;  
 $DF_{Tail}$  ... tail discount factor.

$\{PriorLR_i \mid i = 1, \dots, n\}$  ... prior loss ratio for Bornhuetter-Fergusson method;  
 $\{PriorU_i \mid i = 1, \dots, n\}$  ... prior ultimate estimate for B-F method;  
 $\{BFE_i \mid i = 1, \dots, n\}$  ... expected future amount based on B-F method;  
 $\{BFU_i \mid i = 1, \dots, n\}$  ... total ultimate amount based on B-F method;

## 4.2 TRIANGLES

The relation between an incremental and a cumulative triangle:

$$D_{ij} = \sum_{k=1}^j C_{ik} ; i = 1, \dots, n; j = 1, \dots, n.$$

### Inflation ignored triangles

Incremental triangle with discounted extrapolated values:

$$C_{ij}^{Disc} = C_{ij}^{Bas} * DF_i ; i = 2, \dots, n; j = n - i + 1, \dots, n.$$

Incremental loss ratio triangle:

$$C_{ij}^{LR} = C_{ij}^{Bas} / P_i ; i = 1, \dots, n; j = 1, \dots, n.$$

Age-to-age factors triangle:

$$\lambda_{ij}^{II} = D_{ij+1}^{Bas} / D_{ij}^{Bas} ; i = 1, \dots, n - 1; j = 1, \dots, n - i + 1.$$

Residuals:

$$\begin{aligned}
 r_{ij}^{II,OccPer} &= \lambda_{ij}^{II} - \hat{\lambda}_j^{II,Selected} ; i = 1, \dots, n - 1; j = 1, \dots, n - i; \\
 r_{ij}^{II,CalPer} &= \lambda_{i-j+1,j}^{II} - \hat{\lambda}_j^{II,Selected} ; i = 1, \dots, n - 1; j = 1, \dots, i.
 \end{aligned}$$

## Inflation adjusted triangles

Incremental triangle with nominal values:

$$C_{ij}^{Nom} = C_{ij}^{IA} * IAF_{(i+j-1)-n}; i = 1, \dots, n; j = 1, \dots, n.$$

Incremental triangle with discounted extrapolated values:

$$C_{ij}^{IADisc} = C_{ij}^{IA} * DF_i; i = 2, \dots, n; j = n - i + 1, \dots, n.$$

Incremental loss ratio triangle:

$$C_{ij}^{IALR} = C_{ij}^{IA} / P_i; i = 1, \dots, n; j = 1, \dots, n.$$

Age-to-age factors triangle:

$$\lambda_{ij}^{IA} = D_{ij+1}^{IA} / D_{ij}^{IA}; i = 1, \dots, n - 1; j = 1, \dots, n - i + 1.$$

Residuals:

$$r_{ij}^{IA,OccPer} = \lambda_{ij}^{IA} - \hat{\lambda}_j^{IA,Selected}; i = 1, \dots, n - 1; j = 1, \dots, n - i;$$

$$r_{ij}^{IA,CalPer} = \lambda_{i-j+1,j}^{IA} - \hat{\lambda}_j^{IA,Selected}; i = 1, \dots, n - 1; j = 1, \dots, i.$$

## 4.3 AVERAGE DEVELOPMENT FACTORS

The formulae for average development factors are the same for the triangles with ignored inflation and with inflation adjustment. Thus, for the simplicity, the index representing the inflation manipulation is omitted in this section. Where cumulative triangle data are used as weights, the basic cumulative triangle (Bas) or inflation adjusted cumulative triangle (IA) are used.

Simple average:

$$\hat{\lambda}_j^{Smpl} = \frac{\sum_{i=1}^{n-j} w_{ij} \cdot \lambda_{ij}}{\sum_{i=1}^{n-j} w_{ij}}; j = 1, \dots, n - 1;$$

$$\hat{\sigma}_j^{Smpl} = \sqrt{\frac{\sum_{i=1}^{n-j} w_{ij} \cdot \lambda_{ij}^2}{\sum_{i=1}^{n-j} w_{ij}} - \left(\hat{\lambda}_j^{Smpl}\right)^2}; j = 1, \dots, n - 1.$$

Volume weighted average:

$$\hat{\lambda}_j^{Vwtd} = \frac{\sum_{i=1}^{n-j} D_{ij} \cdot w_{ij} \cdot \lambda_{ij}}{\sum_{i=1}^{n-j} D_{ij} \cdot w_{ij}}; j = 1, \dots, n - 1;$$



$$\hat{\sigma}_j^{Vwtd} = \sqrt{\frac{\sum_{i=1}^{n-j} D_{ij} \cdot w_{ij} \cdot \lambda_{ij}^2}{\sum_{i=1}^{n-j} D_{ij} \cdot w_{ij}}} - (\hat{\lambda}_j^{Vwtd})^2; j = 1, \dots, n-1.$$

LastN<sup>2</sup> factors volume weighted average (for N = 2, ..., 5):

$$\hat{\lambda}_j^{LastN} = \frac{\sum_{i=\max(1, n-N)}^{n-j} D_{ij} \cdot w_{ij} \cdot \lambda_{ij}}{\sum_{i=\max(1, n-N)}^{n-j} D_{ij} \cdot w_{ij}}; j = 1, \dots, n-1;$$

$$\hat{\sigma}_j^{LastN} = \sqrt{\frac{\sum_{i=\max(1, n-N)}^{n-j} D_{ij} \cdot w_{ij} \cdot \lambda_{ij}^2}{\sum_{i=\max(1, n-N)}^{n-j} D_{ij} \cdot w_{ij}}} - (\hat{\lambda}_j^{LastN})^2; j = 1, \dots, n-1.$$

Occurrence period weights – simple average:

$$\hat{\lambda}_j^{OccPerSmpl} = \frac{\sum_{i=1}^{n-j} wOcc_i \cdot w_{ij} \cdot \lambda_{ij}}{\sum_{i=1}^{n-j} wOcc_i \cdot w_{ij}}; j = 1, \dots, n-1;$$

$$\hat{\sigma}_j^{OccPerSmpl} = \sqrt{\frac{\sum_{i=1}^{n-j} wOcc_i \cdot w_{ij} \cdot \lambda_{ij}^2}{\sum_{i=1}^{n-j} wOcc_i \cdot w_{ij}}} - (\hat{\lambda}_j^{OccPerSmpl})^2; j = 1, \dots, n-1.$$

Occurrence period weights – volume weighted average:

$$\hat{\lambda}_j^{OccPerVwtd} = \frac{\sum_{i=1}^{n-j} wOcc_i \cdot D_{ij} \cdot w_{ij} \cdot \lambda_{ij}}{\sum_{i=1}^{n-j} wOcc_i \cdot D_{ij} \cdot w_{ij}}; j = 1, \dots, n-1;$$

$$\hat{\sigma}_j^{OccPerVwtd} = \sqrt{\frac{\sum_{i=1}^{n-j} wOcc_i \cdot D_{ij} \cdot w_{ij} \cdot \lambda_{ij}^2}{\sum_{i=1}^{n-j} wOcc_i \cdot D_{ij} \cdot w_{ij}}} - (\hat{\lambda}_j^{OccPerVwtd})^2; j = 1, \dots, n-1.$$

Extreme values – an individual age-to-age factor  $\lambda_{ij}$  is indicated as an extreme, if:

$$\lambda_{ij} \notin \langle \hat{\lambda}_j^{Selected} - EVP \cdot \hat{\sigma}_j^{Selected}, \hat{\lambda}_j^{Selected} + EVP \cdot \hat{\sigma}_j^{Selected} \rangle;$$

*EVP* ... extreme value parameter.

## 4.4 OPTIONAL DATA TABLES

### Premium

The premium for each occurrence period is simply calculated as a summation of the monthly premium values for respective months.

<sup>2</sup> Simplified notation for Last2, Last3, Last4 and Last5 average types.



## Inflation adjustment factors

The inflation adjustment factors ( $IAF_t$ ) are calculated in three steps. First the inflation rate for each period  $t$  ( $\pi_t$ ) is calculated. Then EdgeFactors ( $EF_t$ ) are calculated from these inflation rates. For the historical (extrapolated) data, this factor represents the inflation adjustment factor for the data from the beginning (end) of the period. And last the inflation adjustment factor for each period is calculated using the geometric average of the factors related to the beginning and to the end of the period (the inflation adjustment factors for the end of the last historical period as well as for the beginning of the first extrapolated period is equal to one by default). To get the analytical expression we consider separately the historical and extrapolated data.

Let us denote  $PS_t$  the first and  $PE_t$  the last day of the  $t$ -th time period. Further consider  $Year(d)$  and  $Month(d)$  as a functions of date  $d$  resulting in the respective calendar year and month (e.g.  $Year(2001-03-31) = 2001$ ,  $Month(2001-03-31) = 3$ ). And last, let  $T$  denote the length of the time period in months (e.g.  $T = 3$  for time period specified as 1/4 Year).

Period inflation rates:

$$Year(PS_t) = Year(PE_t) \Rightarrow \pi_t = \left(1 + \pi_{Year(PS_t)}\right)^{\frac{T}{12}} - 1;$$

$$Year(PS_t) \neq Year(PE_t) \Rightarrow \pi_t = \left[\left(1 + \pi_{Year(PS_t)}\right)^{\frac{12 - Month(PS_t) + 1}{12}} \cdot \left(1 + \pi_{Year(PE_t)}\right)^{\frac{Month(PE_t)}{12}}\right] - 1.$$

PeriodEnd factors:

$$EF_t = \prod_{s=t}^0 (1 + \pi_s); t = -n + 1, \dots, 0;$$

$$EF_t = \prod_{s=1}^t (1 + \pi_s); t = 1, \dots, n - 1.$$

Inflation adjustment factors:

$$IAF_t = \sqrt{EF_t \cdot EF_{t+1}}; t = -n + 1, \dots, -1;$$

$$IAF_t = \sqrt{EF_t \cdot 1}; t = 0, 1;$$

$$IAF_t = \sqrt{EF_t \cdot EF_{t-1}}; t = 2, \dots, n - 1.$$

## Discount factors

The discount factors ( $DF_i$ ) are calculated in two steps. First, the PeriodEnd discount factors ( $PEDF_i$ ) are calculated from the per annum interest rates ( $IR_Y$ ) defined by the user. Here the linear interpolation of the yearly spot rates is used (let  $IR_i$  denote the interpolated per annum

interest rate). Then the development factors applicable to each occurrence period are calculated as a geometrical average of the factors related to the beginning and to the end of the respective period. Regarding the outstanding reserve change the zero duration is assumed, so that the discount factors for the change in reserve and the claims payment booked in one period are the same.

Let us denote  $PS_i$  the first and  $PE_i$  the last day of the  $i$ -th time period. Further consider  $Y(d)$  and  $M(d)$  as functions of date  $d$  resulting in the number of whole years or months between the date  $d$  and the valuation date (e.g. if the valuation date is defined as 1999-12-31, then  $Y(2001-03-31) = 1$ ,  $M(2001-3-31) = 3$ ). And last, let  $T$  denote the length of the triangle time period in months (e.g.  $T = 3$  for time period specified as 1/4 Year).

Interpolated per annum interest rates:

$$IR_i = \frac{12 - (M(PE_i) - 12 \cdot Y(PE_i))}{12} \cdot IR_{Y(PE_i)} + \frac{M(PE_i) - 12 \cdot Y(PE_i)}{12} (IR_{Y(PE_i)} + 1);$$

$$i = n + 1, \dots, 2n.$$

PeriodEnd discount factors:

$$PEDF_i = (1 + IR_i)^{-\frac{T}{12}}; i = n + 1, \dots, 2n.$$

$$PEDF_n = 1.$$

Discount factors:

$$DF_i = \sqrt{PEDF_i \cdot PEDF_{i-1}}; i = n + 1, \dots, 2n.$$

## 4.5 RESULTS

According to the user's choice the results are calculated either from the basic (inflation ignored) or inflation adjusted triangles using the same formulae. For simplicity, the index expressing this choice is omitted in the formulae in this section - assume *Bas* index for the inflation ignored option and *IA* index for the inflation adjusted option (*Disc* and *IADisc* indices for discounted triangle extrapolated values).

Latest incurred:

$$LI_i = D_{i,n-i+1}; i = 1, \dots, n.$$

Triangle ultimate:

$$TrU_i = D_{in}; i = 1, \dots, n.$$

Triangle extrapolated:

$$TrE_i = TrU_i - LI_i ; i = 1, \dots, n.$$

Tail value:

$$Tail_i = (\lambda_{Tail} - 1) \cdot TrU_i ; i = 1, \dots, n.$$

Total extrapolated:

$$TE_i = TrE_i + Tail_i ; i = 1, \dots, n.$$

Total ultimate:

$$TU_i = TrU_i + Tail_i ; i = 1, \dots, n.$$

Triangle extrapolated – discounted:

$$TrE_i^{Disc} = D_{in} - LI_i ; i = 1, \dots, n.$$

Tail value – discounted:

$$Tail_i^{Disc} = Tail_i \cdot DF_{Tail} ; i = 1, \dots, n.$$

Total extrapolated – discounted:

$$TE_i^{Disc} = TrE_i^{Disc} + Tail_i^{Disc} ; i = 1, \dots, n.$$

Prior ultimate for Borhuetter-Fergusson method:

$$PriorU_i = P_i \cdot PriorLR_i ; i = 1, \dots, n.$$

Cumulative development factors from the triangles:

$$CDF_1 = \lambda_{Tail}$$

$$CDF_i = \lambda_{Tail} \cdot \prod_{j=n-i}^{n-1} \hat{\lambda}_j^{Selected} ; i = 2, \dots, n.$$

Expected future amount from B-F method:

$$BFE_i = PriorU_i \cdot (1 - 1/CDF_i) ; i = 1, \dots, n.$$

Ultimate derived amount from B-F method:

$$BFU_i = BFE_i + LI_i ; i = 1, \dots, n.$$

## 5 USEFUL LITERATURE

Basic claims reserving literature:

- [1] **Faculty and Institute of Actuaries.** Claims Reserving Manual. *The Actuarial Profession*. [Online]  
<http://www.actuaries.org.uk/research-and-resources/pages/claims-reserving-manual>.

Statistical calculations:

- [2] **Finch, Tony.** Incremental calculation of weighted mean and variance. *University of Cambridge Computing Service*. [Online]  
<http://www-uxsup.csx.cam.ac.uk/~fanf2/hermes/doc/antiforgery/stats.pdf>