ESG – Yield Curve Simulation

User Guide

CONTENT

1 INTRODUCTION

There are a lot of applications in the financial valuation practice where simulation of future economic scenarios is required. Very often, insurers ask for future interest rates simulations to use them to value the financial options and guarantees offered within the life policies.

This ESG – Yield Curve Simulation tool contains implementation of the Hull-White approach simulating the evolution of future interest rates, very often applied for such purposes.

Traditional simulation of the interest rates evolution may often take very long run-time and that is why this application is offered to help actuaries in their work. By using it you achieve short computation times (e.g. compared to calculation in MS Excel) and it can help you be more effective, save time and simplify the work.

It is assumed that the reader is familiar with or has advanced knowledge of derivative and the interest rate markets.

2 INSTALLATION

The minimum operating requirement is Windows XP SP3 or later. The application requires installation of the Net Framework 4.0 Client Profile. As the Net Framework 4.0 Client Profile is installed through Windows updates, most computers already have it. If not, you should go to http://go.microsoft.com/fwlink/?LinkId=181012 to download the appropriate version.

Once you have downloaded the application from our website www.tools4F.com, browse Windows Explorer for the "Setup" file. Double-click on "Setup.exe" to initiate the setup process. After clicking the information window is displayed. Click "Next" to continue.
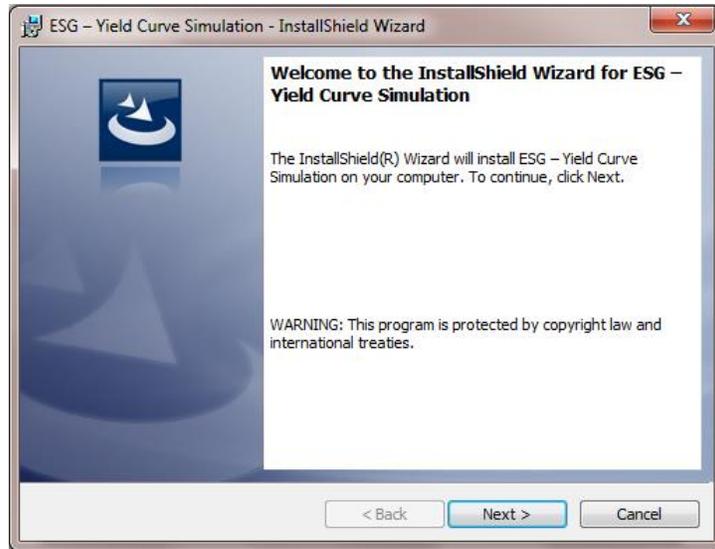
Now you must choose where you would like to install the application. To change the default location click on the "Change" button. Indicate where you would like to have it saved in the file tree. When you are satisfied with the location, click "Next."

You are now ready to begin the installation. Click "Install" to permit the wizard to install the application on your computer.
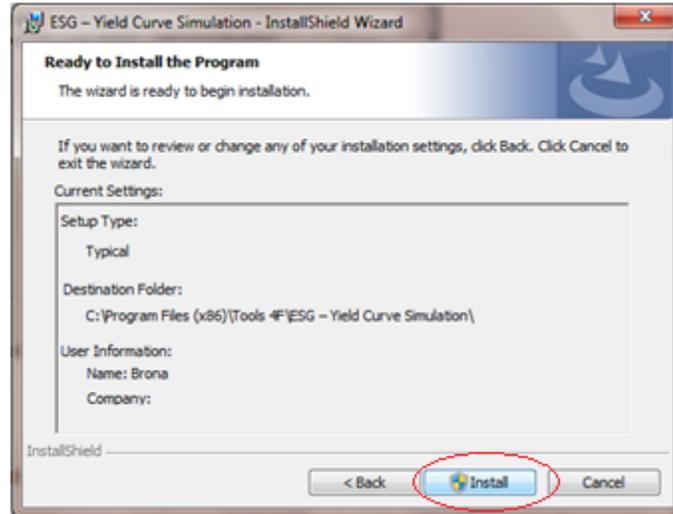
The application icon automatically appears on your desktop.

3   DEMO VERSION AND ACTIVATION

After opening the application, you will be informed about the demo version running. You are free to use the demo version until the full version is activated by entering valid product key. In the demo version you are limited to 5 simulations in one batch and you can also simulate only up to 5 years. Click "OK" to continue.
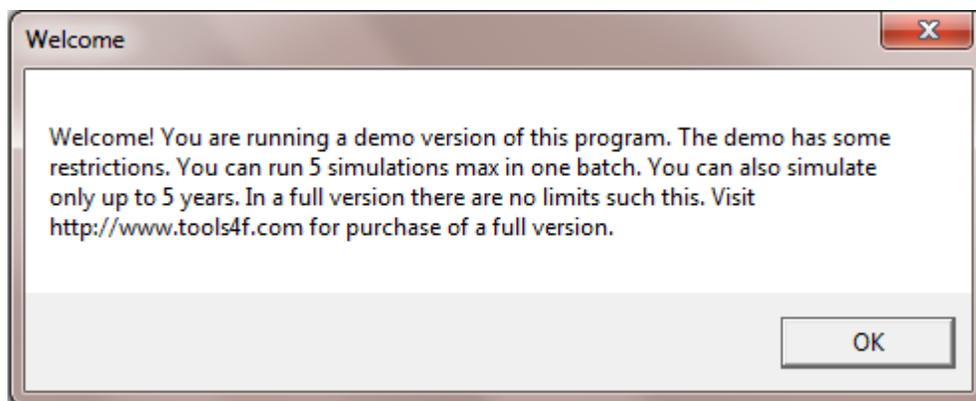
Now only the demo version is active. If you want to run the full version, go to the Main menu bar, select Help menu and choose "Activation".
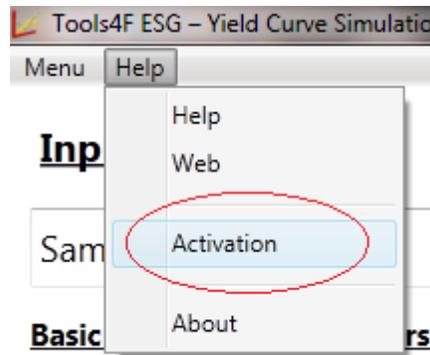
To get the Activation key, go to the web site www.tools4f.com and purchase the application. Once the license is ordered and paid, you will be sent the Activation key to your e-mail. After receiving the Activation key, copy it to the box named "Activation key" and you can activate the full version of this application by pressing the "Confirm" button.

NOTE: The Activation key is generated by means of the Registration key provided by this application and both of the keys are unique to this computer only. You will not be able to use neither the Registration key nor the Activation key on any other computer. Please make sure you are using the correct Registration - Activation key pair, as, once activated on your computer it will not be possible to move and run the activated application on another computer.
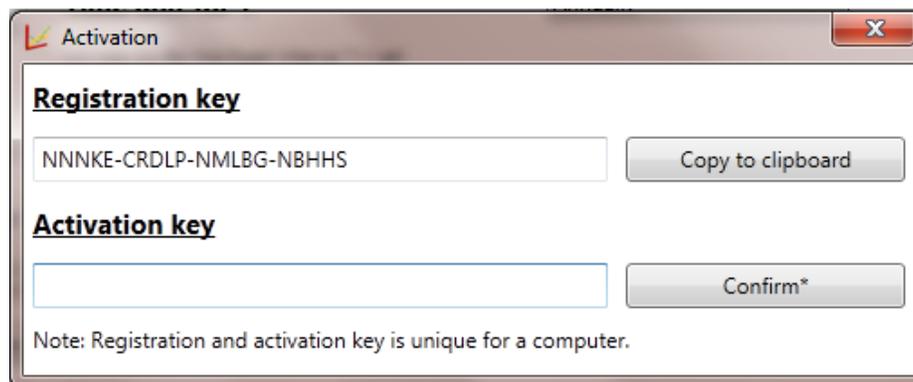
From now on, you can use all the functions of the application.

4   MAIN MENU BAR

The Main Menu bar contains two top-level menu items and each of them has its own list of commands:

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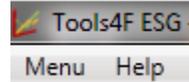
Menu item

- Save configuration – saves the set of rules defining the exact behaviour for almost any window in the application (Basic yield curve parameters - Nelson-Siegel or Svensson, ESG parameters – Hull-White)

- Reload configuration – loads the saved configurations.

- Exit – closes the application.

Help menu item

- Help – automatically opens the manual with instructions how to use this application.

- Web – opens an Internet browser window and directs you to the Tools4F site. You can learn more about our company and our products here.

- Activation – opens the window with the Registration key automatically generated by the system. You can activate the full version here.

- About – opens the window with information such as disclaimer etc.

5   USING THE APPLICATION

The application window is split into three parts: Inputs, Outputs and Graph. Instructions how to use the application will be now explained using an example.

5.1 INPUTS

In the first step, the application offers you to either start with a new configuration or choose one of the saved ones, which allows you to check the previously generated results. The arrows help you to browse through all of the saved configurations. We choose "New configuration" and name it Example.

In the next step the Application requires specification of the parameters of Nelson-Siegel model or Svensson model. Fictional data were used in the example. The parameters of model have to be entered as decimal numbers.

NOTE: You can use our application *Yield curve fitting* for the calculation of Nelson-Siegel or Svensson parameters. For more information see www.tools4f.com.

Basic yield curve parameters - Nelson-Siegel

$\beta 0$	0,041825
$\beta 1$	-0,01387
$\beta 2$	-0,008893
γ(gamma)	3,530323

Once the parameters are set, the parameters for Hull-White model have to be entered into the first two lines: parameter *a* of Hull-White model is a mean reversion parameter and parameter *sigma* of Hull-White model is volatility. The parameters *a* and *sigma* have to be entered as decimal numbers.

NOTE: You can use our application *ESG – Yield Curve Calibration* for the calculation of α and σ parameters. For more information see www.tools4f.com.

Now *T[max] in years* and *Time step for t* should be set. *T[max] in years* means the maximum range of maturity. *Time step for t* means frequency of *t* (projected period). It can attain the following values: Annually, Semi-annually, Quarterly and Monthly.

ESG Parameters - Hull-White

α(alpha)	0,07
σ(sigma)	0,0228083420075726
T[max] in years	50
Time step for t	Annually

Save configurations   Delete configuration

In this area there are buttons to Save or Delete the configuration. After saving the configuration above, the program will inform you that configurations were successfully saved. Click "OK" to continue.

Configurations saved

Configurations were successfully saved.

OK

If you continue by "Delete configuration", the program asks you to confirm. To delete the configuration, click "Yes" in the dialogue box. Click "No" to cancel.

Next you must choose if you want to use either the Internal generator or your own file to generate Random numbers. With the first option the initial number can be set. In case of your own file, click on "From file" and "Browse", it allows you to import the file from your disc. The csv file should contain numbers between 0 and 1 (use the semicolon as delimiter).

The next box in Inputs is for setting the number of simulations. 1000 simulations are chosen in this example.

NOTE: **50 000** is the maximum number of simulations.

Random numbers N(0, 1)

⦿ Internal generator (seed)　　　　　　　　　　　　0

○ From file　　　　　　　　　　　　　　Browse

　　　　　　　　　　　　　　　　　　　No file

Number of simulations　　　1000

You can also choose the option of using antithetic variables by ticking the box under the *Number of simulations*.

Number of simulations　　1

☑ Antithetic variables

5.2 OUTPUTS

This Outputs section allows the user to make specific settings for outputs.

The "Documents" folder is set as default for saving outputs. You may accept the default folder, or modify it as necessary by using the "Browse" button and select another location, The Application will open the browser and the location on your disc can be chosen. The name of the file is also requested in the "Outputs files prefix" box. Outputs files prefix means the first part of the file name (e.g. **Filename**_YtT.csv).

Location

Folder　　　　　　　Browse

C:\Users\Documents

Output files prefix　　　Filename

At this point the setting of Output values is needed. You have the option to choose the value of frequency of *Time unit for T.* Once the drop-down menu opens, you can choose the desired value: Annually, Semi-annually, Quarterly or Monthly.

The Range value must be entered before starting the simulation. The value of the Range influences the display option of the final graph which concerns only output *Y (t, T) for the fixed interval T-t set*.

NOTE: The values of Range are entered without space (e.g. 0,5,10-20).

If the *Time unit for T* is in the value of Year, then the Range indicates the years which you want to display. E.g. If Range is 0, 5, 10 - 20, the years number 0, 5 and 10 - 20 are displayed.

E.g. If *Time unit for T* is Semi-annually, Range is 0, 2, 3, then the first half of the current year, the first and second halves of the next year are displayed.

E.g. If Per year (T) is Months, Range is 0, 12, 15, then the current month, the 12th month and the 15th month are displayed.

NOTE: The years, halves, quarters and months are counted from the present moment.

The Range is not set up for output *P(t, T)* and Y(t, T). The Range is set up automatically for these two outputs as ***T[max] in years x Time unit for T***.

Finally, the user has the option to set "Open result". When the option is checked, the application automatically opens the file(s) with results.

When you are satisfied with the configuration, click "Start simulation."

During the simulation, the progress is displayed at the bottom of the screen. Once the simulation is done, the "Simulation progress" shows 100 %. The simulation speed depends on the performance of the computer. In this example the simulation was done in 3 seconds.

5.3 GRAPH

After the simulation the Graph will appear on the right side of the screen. You have the option to display the graph in period *T-t* that you set up before. The drop-down menu "where T-t" allows you to see the graph in each period that you set up in Range.
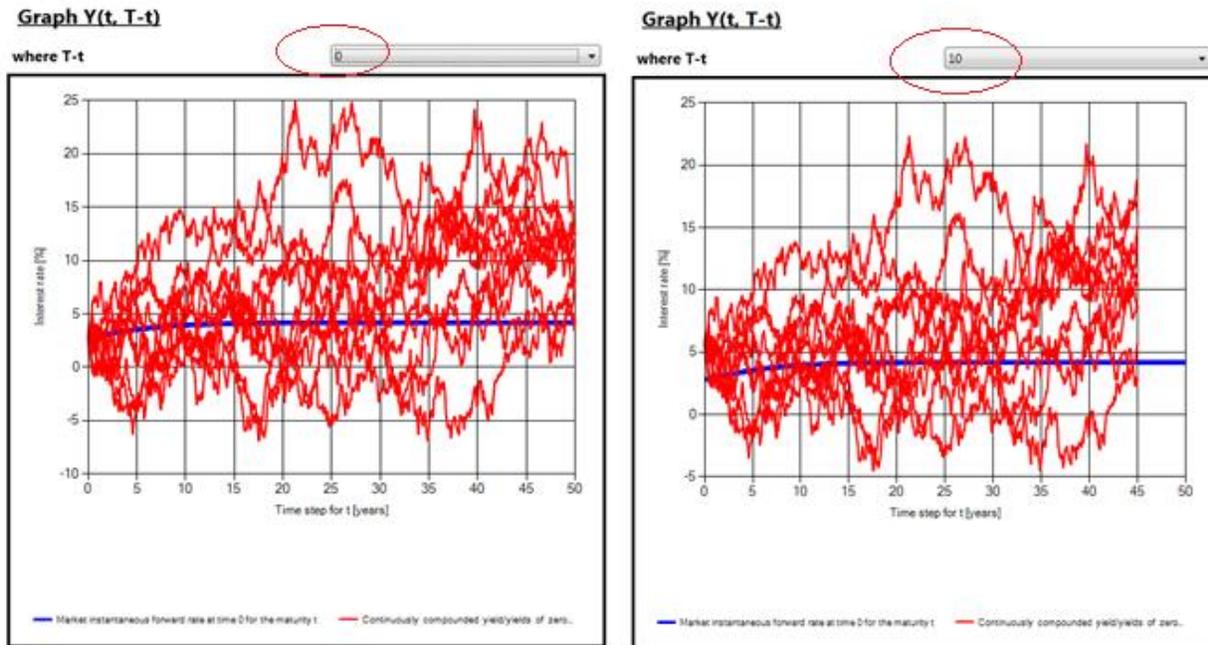
5.4 RESULTS

The Results e.g. *Y(t, T) for the fixed interval T-t set* are automatically saved as *.csv file in the folder which is set up as described above. The results are of the structure which is displayed in the pictures below. The *n* as the number of simulations is defined in the first column, *t* as the number of the month is defined in the second column (as it has been set in our case). It could also be the number of years, semi-annuals or quarters. The *h* in the picture indicates the semi-annual as it was set in *Time unit for T*. It could also be *y* (year), *q* (quarter), *m* (month).

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	n	t	T-t=0h	T-t=1h	T-t=2h	T-t=3h	T-t=4h	T-t=5h	T-t=6h	T-t=7h	T-t=8h	T-t=9h	T-t=10h
2	1	0	0,027955	0,070768	0,068347	0,066185	0,06425	0,062517	0,060962	0,059566	0,058309	0,057177	0,056155
3	1	0,083333	0,024344	0,066247	0,063968	0,061938	0,060128	0,058511	0,057066	0,055773	0,054615	0,053575	0,052641
4	1	0,166667	0,01873	0,05978	0,057675	0,055808	0,054153	0,052683	0,051378	0,050217	0,049185	0,048266	0,047447
5	1	0,25	0,027383	0,067356	0,065181	0,063241	0,061509	0,05996	0,058572	0,057328	0,056211	0,055207	0,054302
6	1	0,333333	0,021847	0,061009	0,059003	0,057222	0,055641	0,054234	0,052982	0,051867	0,050873	0,049986	0,049194

The Results for *P(t, T)* and *Y(t, T)* are displayed just for one set.

6 TECHNICAL DOCUMENTATION

Hull-White model is described by the following equation

$$dr(t) = [\theta(t) - ar(t)]dt + \sigma dW(t) \tag{1.1}$$

where a and σ are positive constants, $W(t)$ is Brownian motion and the function $\theta(t)$ is chosen so as to exactly fit the term structure of interest rates being currently observed in the market. It can be shown that, denoting by $f^M(0, T)$ the market instantaneous forward rate at time 0 for the maturity T, i.e.,

$$f^M(0, T) = -\frac{\partial \ln P^M(0, T)}{\partial T} \ , \tag{1.2}$$

where $P^M(0, T)$ is the market discount factor at time 0 for the maturity T

$$\theta(t) = \frac{\partial f^M(0, t)}{\partial t} + af^M(0, t) + \frac{\sigma^2}{2a}\left(1 - e^{-2at}\right) \tag{1.3}$$

$$r_t = e^{-a}r_{t-1} + f(0,t) - e^{-a}f(0,t-1) + \frac{\sigma^2}{2a^2}\left(1 - e^{-a} + e^{-2at} - e^{-a(2t-1)}\right) + \sqrt{\frac{\sigma^2}{2a}(1 - e^{-2a})}\varepsilon$$

<div align="right">(1.4)</div>

Bond prices at time t with maturity at time T in the Hull-White model are given by

$$P(t,T) = e^{A(t,T) - B(t,T)r_t} ,$$

<div align="right">(1.5)</div>

where

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$

<div align="right">(1.6)</div>

$$A(t,T) = \log\frac{P^M(0,T)}{P^M(0,t)} + B(t,T)f^M(0,t) - \frac{\sigma^2}{4a}(1 - e^{-2at})B(t,T)^2, \quad (1.7)$$

where

a	is the mean reversion rate, a is positive real number
r_t	is the short-term interest rate at time t, formula (1.4)
σ	is positive constant
$f^M(0,t)$	is the market instantaneous forward rate at time 0 for the maturity t
$P^M(0,t)$	is the market discount factor at time 0 for the maturity t

The continuously compounded corresponding investment return are given by

$$Y(t,T) = \frac{B(t,T)}{T-t}r_t - \frac{A(t,T)}{T-t}$$

<div align="right">(1.8)</div>

7 LITERATURE

[1] BRIGO, D., MERCURIO, F.: *Interest rate models*. Springer Finance. Berlin, 2001.

[2] HULL, J. C.: *Options, Futures & Other Derivatives.* Prentice Hall, Upper Saddle River, New Jersey, 2003.

[3] JANEČEK M.: *Valuation Techniques of Life Insurance Liabilities*. Dissertation thesis. MFF UK, Prague.