ESG –Yield Curve Calibration

User Guide

CONTENT

1 INTRODUCTION

There are a lot of applications in the financial valuation practice where simulation of future economic scenarios is required. Very often, future interest rates are generated applying the Hull-White approach (as in the tool ESG – Yield Curve Simulation). However, before the simulation run is started, the Hull-White model parameters have to be determined (typically noted as “α” and “σ”).

The **ESG –Yield curve model calibration** application is designed for Hull-White parameters estimation from the current market values.

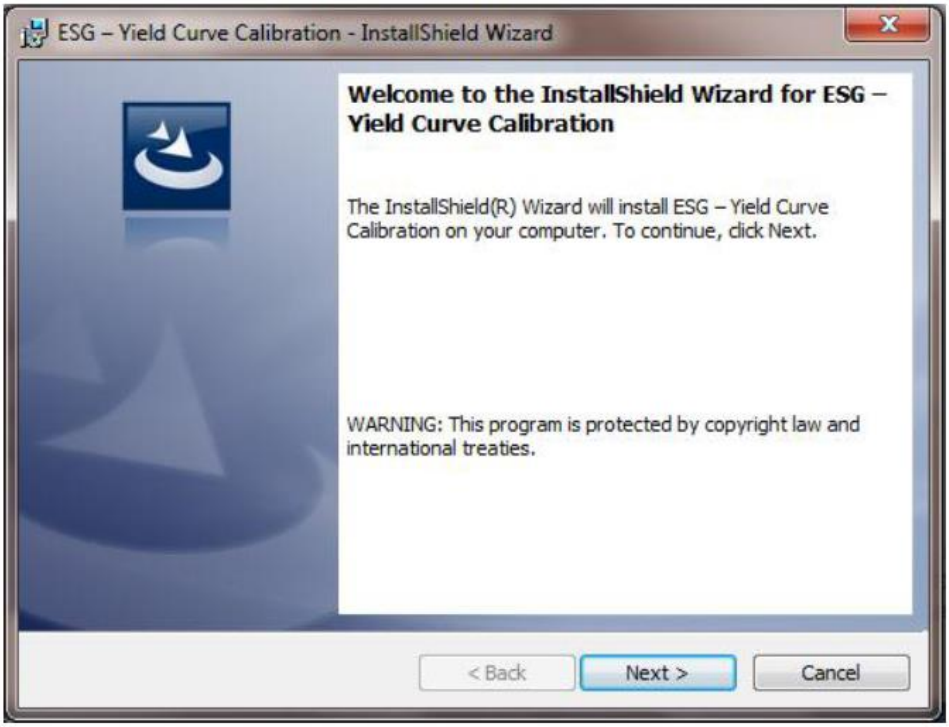
The ESG –Yield curve model calibration allows the user to calibrate to swaptions prices/volatilities – for one swap term or for all swap terms, to calibrate either “α” or “σ”, to display 2D graph (and also to select the values displayed in the graph).

The application ESG –Yield curve model calibration is designed to make your work fast and efficient. Simple and easy to use interface is provided.

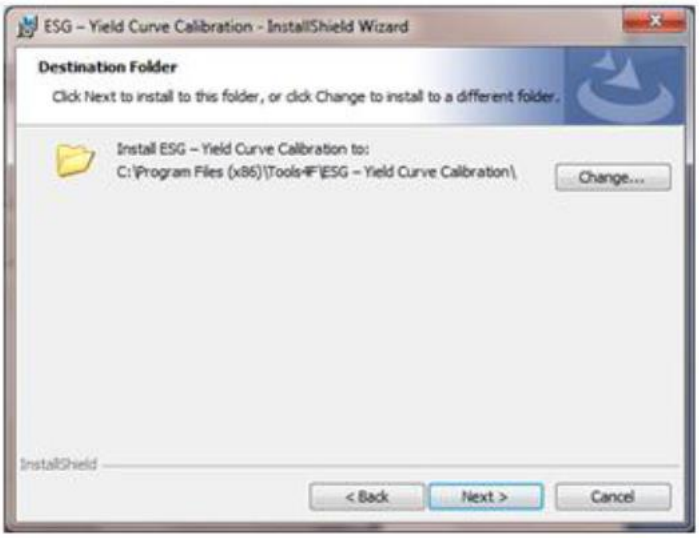
2 INSTALLATION

The minimum operating requirement is Windows XP SP3 or later. The application requires installation of the Net Framework 4.0 Client Profile. As the Net Framework 4.0 Client Profile is installed through Windows updates, most computers already have it. If not, you should go to http://go.microsoft.com/fwlink/?LinkId=181012 to download the appropriate version.

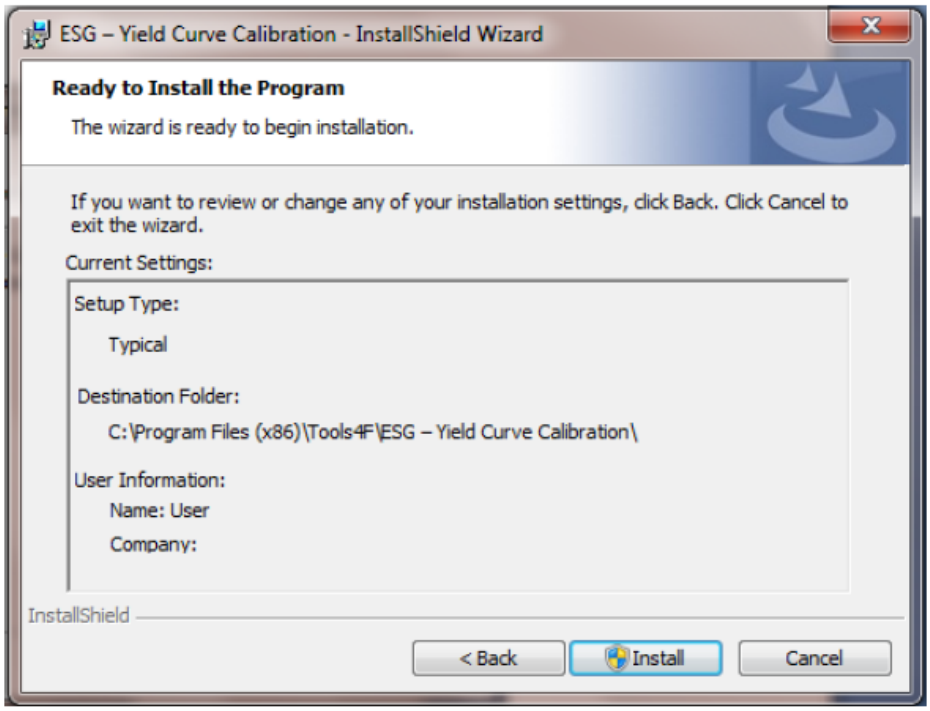
If you have downloaded the application from our web site www.tools4F.com, browse Windows Explorer for the “ESG – Yield Curve Calibration” file. Double-click on “ESG - Yield Curve Calibration.msi” to initiate the setup process. After clicking the information window is displayed. Click “Next” to continue.



Now add your User Name and the name of your Organization. Note, that this information is not required. Next you must choose where you would like to install the application. To change the default location, click on the "Change…" button and select the intended folder. Then click "Next."

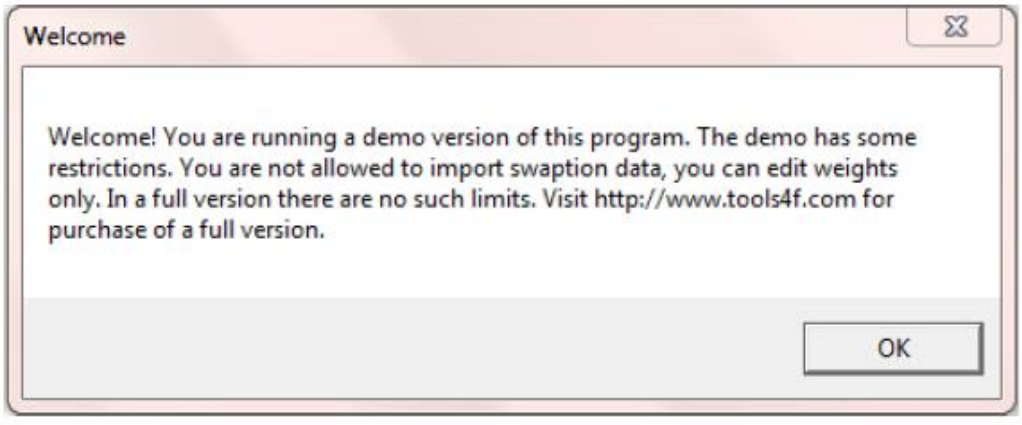


You are now ready to begin the installation. Click "Install" to permit the wizard to install the application on your computer. After finishing the installation, the icon of application automatically appears on your desktop.

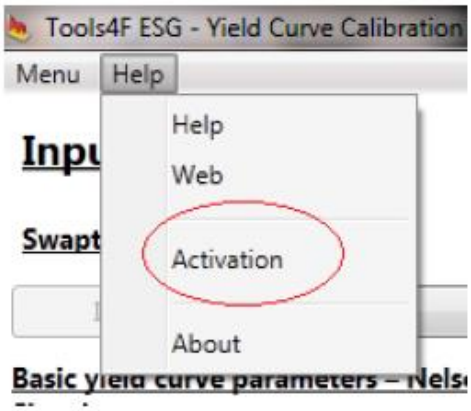


3 DEMO VERSION AND ACTIVATION

After opening the application, you will be informed about the demo version running. You are free to use the demo version until the full version is activated by entering valid product key. In the demo version you can only use the sample swaption data. Click "OK" to continue.

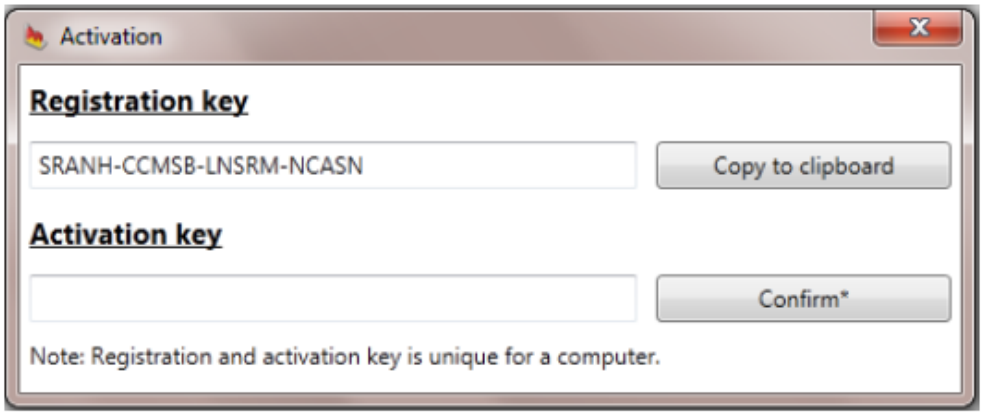


Now only the demo version is active. If you want to run the full version, go to the Main menu bar, select Help menu and choose "Activation".



To get the Activation key, go to the web site www.tools4f.com and purchase the application. Once the license is ordered and paid, you will be sent the Activation key to your e-mail. After receiving the Activation key, copy it to the box named “Activation key” and you can activate the full version of this application by pressing the “Confirm” button.

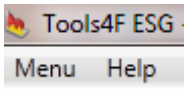
NOTE: The Activation key is generated by means of the Registration key provided by this application and both of the keys are unique to this computer only. You will not be able to use neither the Registration key nor the Activation key on any other computer. Please make sure you are using the correct Registration - Activation key pair, as, once activated on your computer it will not be possible to move and run the activated application on another computer.



4 USING THE APPLICATION

4.1 MAIN MENU BAR

The Main Menu bar contains two top-level menu items and each of them has its own list of commands:



Menu item

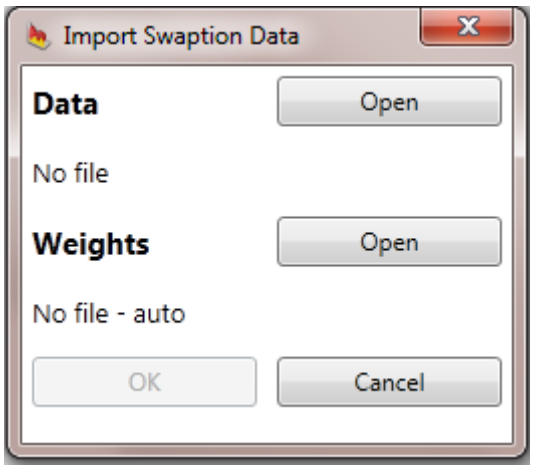
- Import – allows the user to import the market data (swaption volatilities) and their weights in the calculation procedure.
- Edit – allows the user to adjust imported data.
- Exit – closes the application.

Help menu item

- Help – automatically opens the manual with instructions how to use this application.
- Web – opens an internet browser window and directs you to the Tools4F site. You can learn more about our company and our products here.
- Activation – opens the window with the Registration key automatically generated by the system. You can activate the full version here.
- About – opens the window with information such as disclaimer etc.

4.2 INPUTS

At the beginning choose the swaption data by clicking on the button “Import”. It will display the window as follows.



The “Open” button opens the file browser and allows you to select the input data (swaption volatilities) for calculation from your disc. The input data have to meet the following criteria:

- CSV format (use the semicolon as delimiter)
- The header of the table should respect the following structure:
    - Swap term in the first row
    - Option maturity in the column A
- Time in years

Fill in the header according to your data and the intended calculations. Fill in just the maturity which you are interested in and the swap term. The swap term has to be filled for all these maturities. That means that all fields in the rows have to be filled in as shown in the picture below.

NOTICE: It is not possible to make calculation if the input data table includes blank fields.

	A	B	C	D
1		1	2	3
2	0,083333	0,15	0,16	0,17
3	0,25	0,15	0,16	0,17
4	0,5	0,16	0,16	0,16
5	1	0,16	0,15	0,15

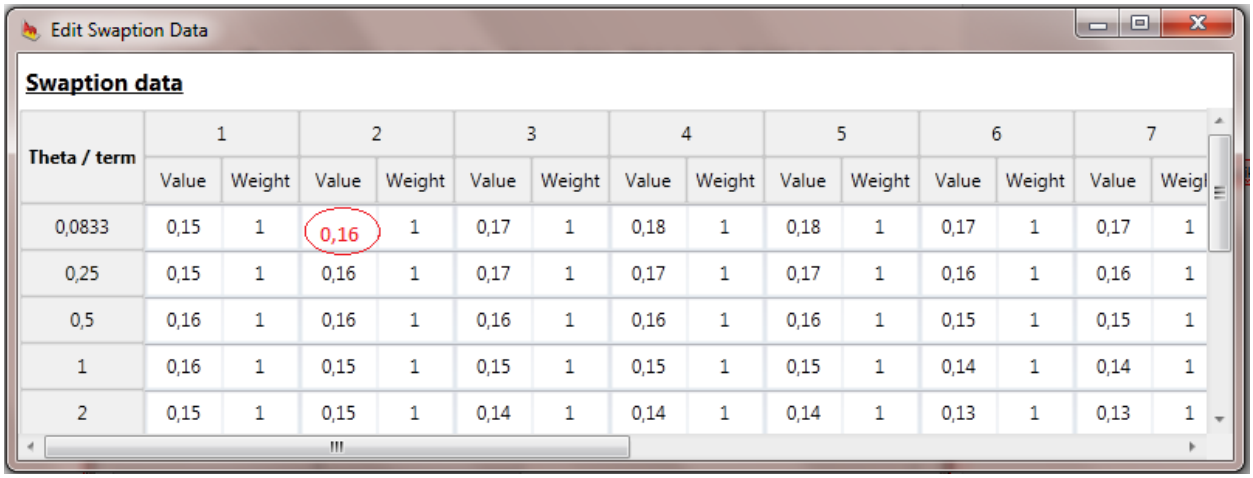
	A	B	C	D	E
1		1	5	10	
2	0,25	0,15	0,17	0,14	
3	1	0,16	0,15	0,13	
4	2	0,15	0,14	0,13	
5					

There is also an option to select the Weights. The Weights indicate the degree of importance for the specific option maturity.

If you do not choose any file, the weights will automatically be set to 1. Once the data and weights are selected, click “OK” to continue.

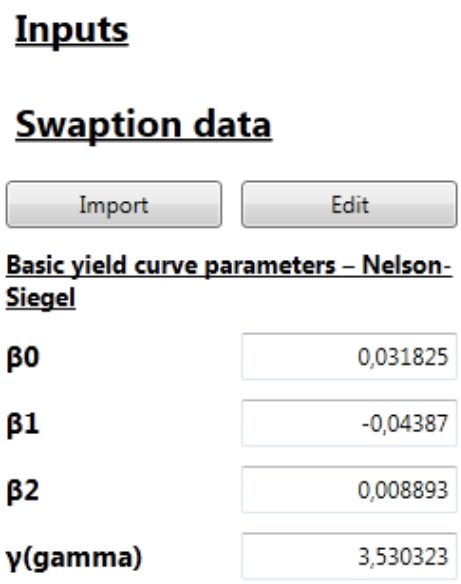
NOTICE: The data and weights have to be in .csv files.

The application offers the option to edit the chosen data. Click on the “Edit” button to display the table with all input data and their weights and adjust them according to your needs. Once you are done editing, click on the red cross in the upper part of the window.



In the next step the Application requires the basic yield curve parameters - parameters of Nelson-Siegel model to be specified. The parameters of Nelson-Siegel or Svensson model have to be entered as decimal numbers. Once you enter a wrong value, the "Calibrate" button becomes inactive.

NOTICE: You can use our application ***Yield curve fitting*** for the calculation of Nelson-Siegel parameters. For more information see www.tools4f.com.

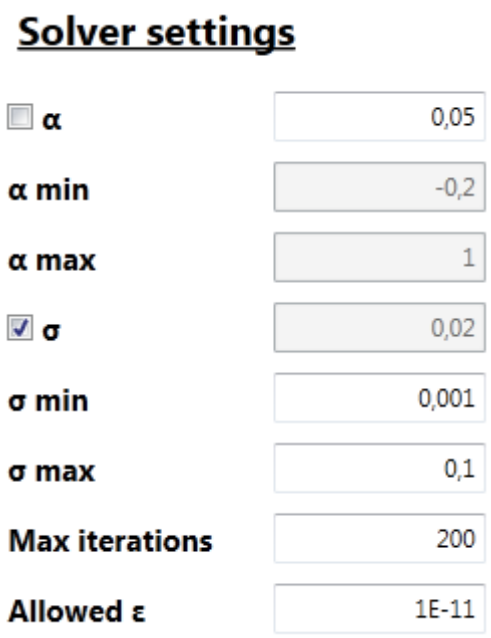


In the “Solver settings” area, you are asked to specify the result parameters α and σ. The checkbox placed next to the parameters allows you to set these parameters as fixed or set them within the limits of the interval.

Once the checkbox is checked, the parameter is found by means of a numerical procedure within the limits of the interval, otherwise the parameter is equal to the set value.

For the best results which suit your needs, the application offers the possibility to set the maximal number of iterations and also the maximal value allowed (Ɛ). The application runs until the difference between the consecutive SSE is less than Ɛ.

NOTICE: Please, be aware that the result can be very much affected by the set value of allowed Ɛ. The value too high or too low may cause wrong results. The default value should be 1E-10.

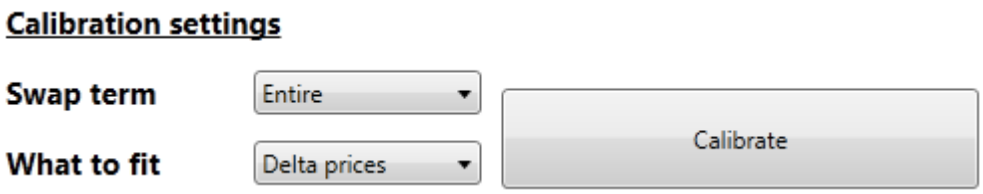


The Calibration settings section allows you to choose specific setting for the calibration procedure. The first option is to set the “Swap term”. Once the drop down menu opens, you can choose the value “Entire” or you can choose every single maturity which you have set as inputs.

Next you set “What to fit”. There are two possibilities what to fit: Delta prices, Delta volatilities. Once the Delta prices are fitted, the smallest sum of squares of market prices as well as fitted

prices is searched for. Once the Delta volatilities are fitted, the smallest sum of squares of market volatilities and price volatilities is searched.

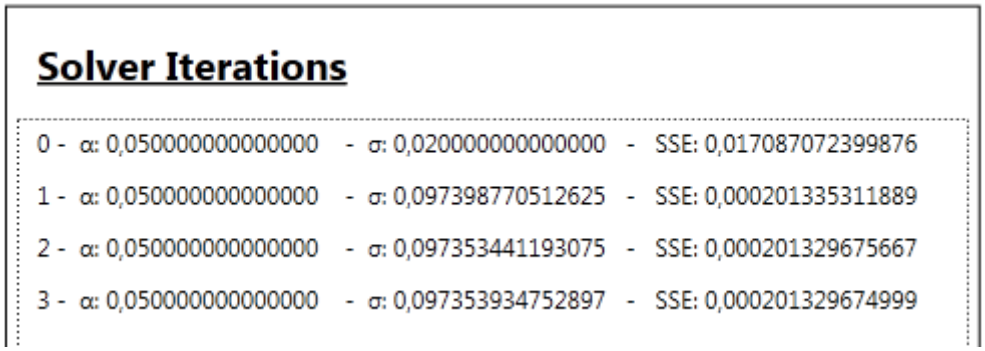
To start the calibration, click on the "Calibrate" button.



NOTE: If any cell is filled-in incorrectly, the "Calibrate" button is inactive.

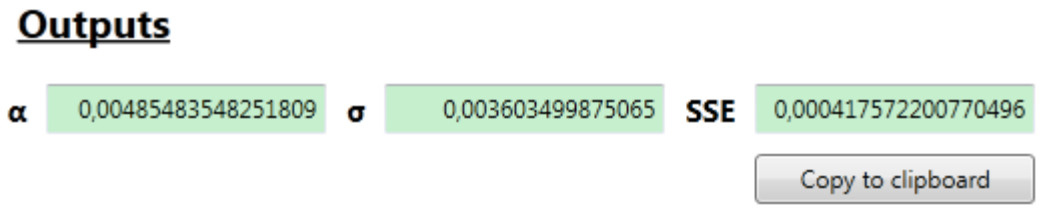
4.3 OUTPUTS

Once the calibration starts, you can see the partial result of calibration in the Solver Iterations window.

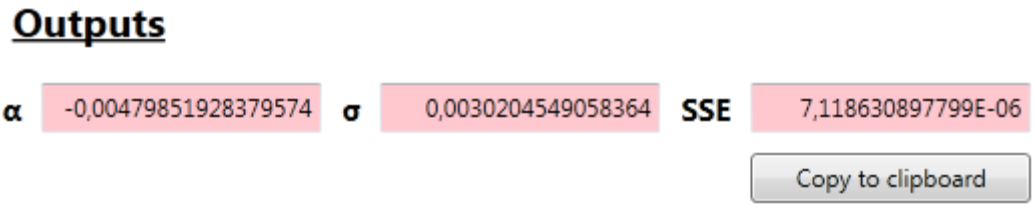


The resulting parameters are displayed in the upper part of the screen after the calculation is finished. For the most convenient work with the application, the "Copy to clipboard" button is placed next to the results. This button allows you to copy the results α, σ and SSE and work with them in another document (e.g. the results will be copied as a column in Excel).

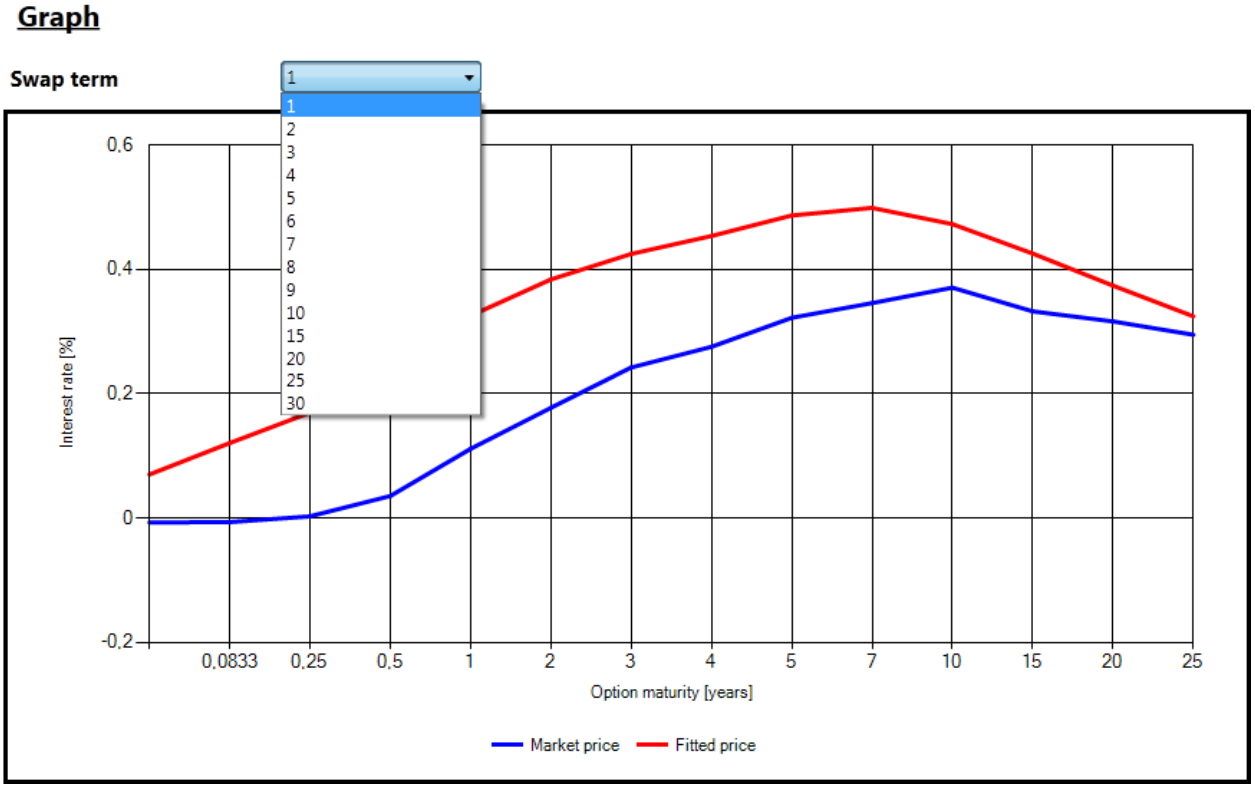
NOTICE: You can use these resulting parameters as the inputs parameter to the application for simulation of interest rates ***ESG – Yield Curve Simulation***. For more information see www.tools4f.com



If the Outputs are in pink cells, it indicates that the results found are not optimal. It can be caused by unsuitably chosen value of Allowed Ɛ (described above).



After the calibration is finished a graph will appear on the right side of the screen. The graph displays the comparison of two curves - market price/volatilities (by Black-Scholes) and fitted price/volatilities (by Hull-White). You can display a specific graph for each swap term in case you have chosen the swap term “Entire” within the settings of parameters.



5 TECHNICAL DOCUMENTATION

Market Price – Black-Scholes

The standard market model gives the value of a swaption where the holder has the right to pay s_K as

$$BS\ Swaption\ =\ \sum_{i=1}^{mn}\frac{L}{m}P^M(0,T_i)[s_0N(d_1)-s_KN(d_2)], \tag{2.1}$$

where

$$d_1=\frac{ln\left(\frac{s_0}{s_K}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}, \tag{2.2}$$

$$d_2=d_1-\sigma\sqrt{T}, \tag{2.3}$$

L	is the notional principal, assumption $L\ =\ 1$,
m	is the number of payments per year, assumption $m\ =\ 1$,
$N(\cdot)$	is the cumulative distribution function of standard normal distribution,
$P^M(0,t)$	is the market discount factor at time 0 for the maturity t,
σ	is the volatility of the forward swap rate,
T	is the time to maturity of the option,
$Term$	is the swap term,
s_K	is the strike price,
s_0	is the forward swap rate at time zero.

$$s_0=\frac{P^M(0,T)-P^M(0,T+Term)}{\sum_{i=1}^{mn}P^M(0,T_i)} \tag{2.4}$$

The $\sum_{i=1}^{mn}P^M(0,T_i)$ term is the discount factor for the mn payoffs. Defining A as the value of a contract that pays $1/m$ at times $T_i(1\le i\le mn)$, the value of the swaption becomes

$$LA[s_0N(d_1)-s_KN(d_2)], \tag{2.5}$$

where

$$A=\frac{1}{m}\sum_{i=1}^{mn}P^M(0,T_i), \tag{2.6}$$

$$P^M(0,t)=e^{-Y(0,t)\cdot(T-t)}, \tag{2.7}$$

where Nelson-Siegel formula for spot continuously compounded yield curve is

$$Y(0,t) = \beta_0 + (\beta_1 + \beta_2) \cdot \frac{\left(1 - e^{-t/\gamma}\right)}{t/\gamma} - \beta_2 \cdot e^{-t/\gamma}, \tag{2.8}$$

where

$\beta_0, \beta_1, \beta_2, \gamma$ Nelson – Siegel parameters

Hull-White Price

Suppose we work in the HJM (Heath-Jarrow-Morton) one factor model with a volatility term. We use the notation $c_i = \delta_i R \ (1 \leq i < n)$ and $c_n = 1 + \delta_n R$. The price of an European receiver swaption, with expiry T and strike rate R on a swap with starting date t_0 and maturity date t_n, is given at time 0 by

$$\sum_{i=1}^{n} c_i P^M(0,t_i) N(\kappa + \alpha_i) - P^M(0,t_0) N(\kappa + \alpha_0), \tag{2.9}$$

where κ is the (unique) solution of

$$\sum_{i=1}^{n} c_i P^M(0,t_i) exp\left(-\frac{1}{2}\alpha_i^2 - \alpha_i \kappa\right) = P^M(0,t_0) exp\left(-\frac{1}{2}\alpha_0^2 - \alpha_0 \kappa\right) \tag{2.10}$$

and

$$\alpha_i^2 = \frac{\sigma^2}{2a^3}(e^{-aT} - e^{-at_i})^2(e^{2aT} - 1) \tag{2.11}$$

$P^M(0,t_i)$ is the market discount factor at time 0 for the maturity t, formula (2.7)

Implied Volatility

For the calculation of implied volatility will be used The Newton - Raphson Iteration [5], i.e.:

$$Vol_{i+1} = Vol_i - \frac{BS\ Swaption_i\ -\ HW\ price}{BS\ Swaption\ Vega_i} \tag{2.12}$$

where

$BS\ Swaption$ is formula (2.1)
$HW\ price$ is constant, formula (2.9)
$Vol_i,\ i = 0$ is predefined constant

$$BS\ Swaption\ Vega = (BS\ Swaption)' =$$

$$P^M(0,T_i)\left[s_0 \cdot \frac{exp\left(-\frac{d_1^2}{2}\right)}{\sqrt{2\pi}} \cdot \left(\frac{\frac{\sigma^2}{2}T - ln\left(\frac{s_0}{s_K}\right)}{\sigma^2\sqrt{T}}\right)\right] - \left[s_K \cdot \frac{exp\left(-\frac{d_2^2}{2}\right)}{\sqrt{2\pi}} \cdot \left(\frac{-\frac{\sigma^2}{2}T - ln\left(\frac{s_0}{s_K}\right)}{\sigma^2\sqrt{T}}\right)\right] \quad (2.13)$$

This recurrent calculation is repeated until the

$$ABS(BS\ Swaption - HW\ price) > Tolerance \quad (2.14)$$

where

$Tolerance$ is predefined constant

6 LITERATURE

[1] BRIGO, D., MERCURIO, F.: *Interest rate models*. Springer Finance. Berlin, 2001.

[2] HENRARD M.: *Explicit Bond Option and Swaption Formula in One Factor Heath – Jarrow – Merton Model.* International Journal of Theoretical and Applied Finance 6, 2003, p. 57–62.

[3] HULL, J. C.: *Options, Futures & Other Derivatives.* Prentice Hall, Upper Saddle River, New Jersey, 2003.

[4] The Newton - Raphson method. *Mathematics Department* [online]. 2011 [cit. 2012-02-09]. Dostupné z: http://www.math.ubc.ca/~anstee/math104/104newtonmethod.pdf